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ABSTRACT

This guide to mathematics education is designed to provide direction for teachers and administrators in the development and evaluation of school mathematics programs. The skills and knowledge of mathematics are seen by the Hawaii Department of Education to be of significant importance in our society, with the Department assuming primary responsibilities for developing programs which will help all Kindergarten through grade 12 pupils to become effective and contributing members of society. Sections include: (1) an introduction, explaining the purpose of the guide, the mathematics program, and program highlights: (2) the learning environment, in terms of the school, teacher, and classroom: (3) program goals for Kindergarten through grade 12, examining both broad and procedural goals, the need for problem-solving instruction, and a detailed table of problem-solving behavior: (4) curriculum guidelines, including an intricate scope and sequence overview, performance expectations, individual curriculum guidelines for each year in the Kindergarten through grade 6 sequence, and an outline of how learner objectives are related to performance objectives in the elementary program: and (5) a bibliography. (MP)

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grades k-6 mathematics program guide

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FOREWORD

The skills and knowledge of mathematics continue to be of significant importance in our society. Toward this end, the Department of Education has primary responsibilities for developing programs which will help all K-12 students to become effective and contributing members of society. This guide is an effort to provide direction for a mathematics program which allows all students to reach their full potential in the area of mathematics. An initial draft developed by teachers with technical guidance provided by the University of Hawaii was disseminated for field review during Spring 1978. The thoughtful consideration of teacher and curriculum specialist reviewers is reflected in this revised version of the Mathematics Program Guide.

We hope that all teachers and principals will find this guide useful in directing improved mathematics programs in their classes and schools.

Charles G. Clark
Superintendent of Education

JAN 2 6 1981

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CHAPTER I

INTRODUCTION

Purpose of the Guide

The purpose of this Guide is to provide direction for teachers and administrators in the development of school-level mathematics programs and in the evaluation of those programs. The Guide is to build a base from which schools can develop adequate objectives for their programs. Furthermore, these guidelines can be useful for those responsible for the selection of materials and experiences for pupils, and for those responsible for the education of mathematics teachers. It is expected that the Guide will be reviewed at regular intervals.

The Mathematics Program

The mathematics program is that totality of ingredients that provides for the mathematics education of each pupil to the pupil's potential. This program first directs its efforts to produce citizens who are mathematically functional in everyday living. At the same time, because not all types of mathematical problems requiring solution in the future are presently known, the program must be rich in problem-solving experiences. Such a program would guide pupils in developing strategies for attacking the problem, reasoning effectively using mathematics as a tool, and resulting finally in a solution or some other valid decision. In general, then, pupils must develop certain skills, abilities, and attitudes that will provide a basis through which they "learn to learn" after they have left the guidance of the school.

This Guide identifies the pupil as the central figure in the educational scene. At the same time it identifies the effective teacher as the one most responsible for facilitating maximum learning by the pupil. But the Guide also identifies the optimum program as one with pupils, teachers, school administrators, and the public working harmoniously in a common purpose--providing the best possible mathematics education for all.

The state, district, and school levels share responsibilities in providing for the mathematics education of students. The responsibilities at the state level include the identification of mathematical needs of students and the planning and development of mathematical goals, curriculum objectives, and related educational policies for Statewide application and administration. At the district level responsibilities include assistance in the implementation of the mathematics program through interpretation, articulation, consultation and monitoring. The primary responsibility at the school level is instruction to meet the goals and objectives of the mathematics program.

Highlights of the Mathmatics Program Guide

The Mathematics Program Guide emphasizes 1) communication in the classroom; 2) delay in the introduction of certain symbolism, technical language and topics; 3) metrics and the earlier introduction of decimal fractions; 4) provisions for wider educational opportunities; 5) instruction in mathematical reasoning processes; 6) instruction in basic skills; and 7) wider usage of physical models and pictures. Each of these are described below.

1. The importance of communication. In the past mathematics educators have concentrated on the communication of mathematical facts, ideas, and procedures by having students listen and read. These aspects remain very important because they must listen, read textbooks, newspapers, magazines,

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reports, and so on critically. Equally important, however, for all people is the ability to express verbally and in writing mathematical concepts in order to live effectively in a technological society. A further need is the ability to ask searching questions about quantitative and spatial situations in order to secure needed answers to clarify one's own thinking.

2. The importance of delaying certain symbolism, certain technical language, and certain topics. The focus on problem solving demands that students deal with ideas. Too frequently in the elementary grades, ideas become vague when technical terminology and symbolism are introduced. The Guide provides for a more gradual introduction of technical names, symbolism, and formalization of mathematical relationships.
3. The importance of the metric system and decimal fractions. Hawaii's public schools are committed to the use of the metric system. Throughout the Guide the examples given are primarily in metric units. While the emphasis is on the metric units, important customary units should be taught. Because of the use of decimals in converting metric units, decimals are introduced earlier than presently done. Coupled with this is the delay in teaching some of the fraction algorithms.
4. The importance of providing wider educational opportunities. The Guide accommodates the varying needs and interests of students in the high school. A full, 4-year sequence of study in mathematics in available to every student. Three options are available for students with courses designed for flexibility and mobility within and among options. All options contain the basic essentials of mathematical literacy and provide for the attainment of essential competencies of high school graduates.

Because of their limited effectiveness, the traditional General Mathematics courses are deleted. In their place are the various courses in Option X as described in the section on Curriculum Guidelines, 9-12.

The Guide provides wider opportunities for more students to explore algebraic and geometric concepts which can result in extended educational and career options.

5. The importance of mathematics reasoning processes. Discovering relationships, making and checking conjectures, giving examples and counter-examples, applying past knowledge to new situations, and presenting mathematical arguments are examples of reasoning processes that should be developed and maintained. Students must be afforded the time for such development and maintenance. This necessitates careful selection of the content areas to be studied. Mastery in mathematics implies more than knowing concepts; it also includes the reasoning processes. The Guide reflects this view by providing fewer content areas than might be normally expected since attempting to cover too many content areas tends to eliminate the fostering of these reasoning processes. Essential content areas are maintained.

6. The importance of basic skills. Basic skills is given a broader interpretation than merely basic computational competence in this Guide. Such competence is necessary but not sufficient for mathematical literacy. The definition of basic skills used in this Guide incorporates appropriate computational skills as well as skills in measuring, estimating, determining reasonableness of results, and reading, constructing and interpreting tables and graphs.

The National Council of Teachers of Mathematics has expressed this view in a position paper which stresses the need a) to know when to multiply as well as how to multiply and b) to develop geometric intuition and to interpret data. (See Appendix A.) Further, the National Council of Supervisors of Mathematics has listed ten basic skills of which appropriate computational skills is one. (See Appendix B.)

7. The importance of physical models and pictures. The need for physical representations and pictures for mathematical ideas and procedures has been recognized for a long time. It is well documented by research and by effective classroom teachers who report a higher degree of skill, deeper understanding, and more favorable attitudes of pupils who have learned through such a program.

CHAPTER II

THE LEARNING ENVIRONMENT

The learning environment includes the school, the teacher, and the classroom. Each contributes to the success of the mathematics program.

The School

An effective mathematics program requires the coordinated efforts of all schools responsible for the mathematics education of students. It is essential that intermediate schools articulate with their feeder elementary schools, and that high schools articulate with their feeder intermediate schools. Equally important is the need for the high schools to communicate with institutions of higher learning.

To attain the mathematics program goals each school, elementary, intermediate, and high, has a major role to play. A description of the role of each school follows.

The Elementary School. The role of the elementary school (K-6) is to provide a basic, firm foundation in mathematics that will enable students to achieve their potentials in mathematics. Included in this basic foundation are:

1. the acquisition of knowledge and understandings, and the development of concepts and skills in arithmetic, geometry, and measurement.
2. the development of provlem-solving techniques, logical and independent reasoning, and confidence in one's ability to apply mathematics to the environment.
3. the awareness and appreciation of one's needs, values, interests, and achievements as related to mathematics.

4. the development and sustenance of the ability and desire to inquire, explore, investigate, estimate, and solve realistic everyday problems which the student faces.

The Intermediate School. The intermediate school has two major responsibilities in the attainment of the mathematics program goals. The manner and extent to which these responsibilities are carried out depends on varying individual characteristics of the student population.

The first responsibility relates to the broad concept of "basic skills." Those skills which have been developed in the elementary school must be maintained and extended to applications in the real world outside the classroom. The basic skills are not restricted to computational procedures, but are also related to personal and social needs. They, therefore, include applications to consumer activities, use of mathematics in communications (verbally, symbolically, graphically), estimation procedures, analysis procedures with regard to compiled data, reasoning processses leading to and from generalizations, basic geometric concepts as related to our three-dimensional world, and increased emphasis on the metric system of measurement. The highest level of functional, quantitative thinking is that done in connection with the solving of problems that are vital and realistic to the learner and are clarified by mathematical techniques.

The second responsibility deals with the nature of mathematics itself. Various selected topics from the realm of mathematics are introduced to exhibit the vast range of thought involved in one of civilization's greatest productive achievements. Elements of the history of the development of mathematics and their effect on civilization are explored, as is the role of the computer in today's society. Selected ideas from the areas of number theory, algebra, descriptive and coordinate geometry, trigonometry, and topology are introduced to spur interest in the further study of mathematics as a useful, growing, pleasurable area of human endeavor.

Within these two responsibilities lies the understanding that the intermediate program will provide the background for the successful study of algebra in the high school for many of its students, and may provide the algebra course itself in the eighth grade for highly capable students.

The High School. The role of the high school is to provide a background in mathematics that will enable students to continue their academic pursuits in colleges or technical schools, to provide a mathematics background sufficient for employment, and to provide the mathematical competence necessary to function adequately in society. In order to fulfill this role, the high school must:

1. offer a broad selection of courses. These courses must provide opportunity and encouragement for all students to study mathematics throughout the high school years. Mathematically talented students may be able to proceed through the traditional sequence of courses ending with a course in trigonometry or calculus.
2. encourage greater flexibility between and within program options. The provision for semester courses rather than year courses will assist in this effort. In addition students should be able to take more than one mathematics course at a time.
3. provide academic counseling. The mathematics teachers and counselors must communicate clearly and accurately the many options available in the selection of courses and the many changes in the needs and requirements in job training and college programs.

4. provide for experimentation. The experimentation of new and hopefully better approaches to teaching and learning should be encouraged.

The Teacher

Teachers are key figures in the mathematics education of students. As such they have numerous roles to play. These are described below.

Individual Commitment. The mathematics program at any level is no better than the teachers who are responsible for supporting and implementing the program at the class level. To guarantee any measure of success, it is important that teachers thoroughly acquaint themselves with the Mathematics Program Guide and make a commitment to follow through with its ideas and work toward its continuing improvement.

Communication. Teachers must be able to communicate with each other, students, administrators, parents, and the lay public if there is to be support for cooperation and accomplishing the goals of the program.

Mathematical Competencies. Teachers' mathematical competencies should be updated continually by studying mathematics. Further, their knowledge of and ability to present applications of mathematics such as to calculators, consumer education, and computers should be kept current. It is equally important that teachers be prepared to teach all students: those whose formal education may be terminated at the high school level, those who may have learning difficulties in mathematics for various reasons, and those whose formal education may extend beyond the high school.

Guidance and Placement. Mathematics teachers should have a major role in the counseling of students into mathematics classes, communicating the many changes in the needs and requirements in job training and college programs. (See Appendix C.) Mathematics departments in grades nine through twleve should develop systematic counseling programs and procedures for all mathematics students.

Professional Improvement. It is the responsibility of each teacher to contribute to the quality of mathematics education through personal involvement and participation in in-service programs, curriculum development, and evaluation.

Effective Classroom Teachers. While every teacher has made a commitment to guide pupils toward steady growth and the greatest possible achievement, it is worth reviewing on at least a semi-regular basis what that commitment involves. In general, effective teachers are willing to do everything possible to instill a sense of worth and success in each student. To achieve this, the teachers need to be aware of each student's needs; provide students with attainable goals; show respect, tact, and warmth toward them; and maintain a two-way communication with them.

The Classroom

The most effective and efficient climate and environment for learning provide for the following:

1. an inductive sequence of experiences that makes it possible for the learner to bridge the gap from the concrete to the abstract.
2. a means of communication that the learner can understand.
3. opportunities for learners to become involved in appropriate activities.
4. opportunities for the teacher to study the learner's habits of work and reasoning processes.

5. motivation for learners to continually improve their proficiencies in mathematical skills and concepts.

The development of an optimum climate and environment for the teaching and learning of mathematics is no easy matter for there are many obstacles to overcome. However, the attainment of the majority of the provisions listed above might mean the difference between a successful mathematics program and a mediocre one. Therefore, every effort towards their attainment should be made. Success could result from the development or acquisition of pertinent materials for the classroom, growth in techniques of open communication, or even rearranging the classroom.

Since mathematics is often highly abstract, it is important to provide experiences that will make symbols meaningful to students. Manipulative materials provide effective means for facilitating learning. Manipulating may mean handling an object, comparing objects, viewing objects represented in a pictorial mode, or engaging in paper-and-pencil activities. The materials should provide a smooth transition from concrete learning experiences to the abstract.

Instructional materials should be flexible enough to be used with different teaching methods and organizational plans. Rates of learning will vary and the pacing of instruction should be planned accordingly. Perhaps of more significance, the pupils' modes of thinking will differ--some think best in concrete terms, others in abstract formulations. Some deal best with pictures and others with words or symbols.

The establishment of classroom climate, under the direction of the teacher, should be pupil-oriented and non-threatening. Using defined instructional objectives, the teacher assumes the role of a guide who directs

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learners to explore, investigate, estimate, and solve everyday, realistic problems.

The climate of the classroom should provide for an atmosphere of open communication, interaction among pupils, and a spirit of discovery. Questions and problems from students should be encouraged. Problems should be solved in a variety of ways and solutions accepted in many different forms. Interaction should be encouraged, because students learn from one another and mathematics becomes a vibrant, vital subject when points of view are argued.

A significant feature of a mathematics learning environment is the spirit of free and open investigation. The learning of mathematics is many-faceted. Pupils and their teachers must feel free to express and explore those facets that have particular meaning for them. The classroom environment is an important but often overlooked facet. It should be organized and equipped to appear as a laboratory for learning and should relate learning to past experiences while providing new experiences as needed. Well-equipped and organized classrooms allow pupils to share the responsibility for their own learning and progress.

CHAPTER III

MATHEMATICS PROGRAM GOALS, K-12

In this chapter, the broad goals of the mathematics program are first discussed. This is followed by a set of procedural goals that facilitate the attainment of the broad goals. Because problem solving is a critical goal of the mathematics curriculum it is singled out for additional discussion.

Broad Goals

Broad goals are those that the program strives to achieve throughout the K-12 mathematics curriculum.

1. Develop mathematical competence to function effectively in today's society.

   Mathematical competence includes knowledge and application of mathematical facts and spatial properties and relationships, proficiency in mathematical skills, and successful operation in economic situations. It extends from dealing smoothly with everyday encounters with mathematics to satisfying career-oriented needs.

2. Develop understanding of the importance and relevance of mathematics historically and in the world today.

   This goal includes awareness of the nature of mathematics and its place in our culture, recognition of its richness and its power, and understanding the uses and limitations of modern technological advances such as computerization.

3. Develop ability to think critically and to solve problems.

   This encompasses not only the development of more formal techniques and strategies for gathering and analyzing data, reasoning logically, making judgments, communicating ideas through appropriate symbolism, and validating results, but also encouraging intuitive, creative, and uniquely-individual approaches and resolutions to problems.

4. Nurture intellectual curiosity and promote the desire to continue learning.

   This includes fostering sensitivity to and inquisitiveness about the environment, observing, questioning, and exploring with a "free and confident attitude," and meeting new situations with resourcefulness and enthusiasm.

Procedural Goals

Procedural goals are those that facilitate the achievement of the broad goals.

1. Provide for individual differences as all students cannot be expected to reach the same standard at the same time:
    a. respect the emotional, physical, and individual needs of students from all cultures, socio-economic levels, family structures, and diverse backgrounds.

    b. provide a wide variety of experiences in each class so each pupil, whether in the introductory, developmental, refinement, reinforcement, or enrichment stage will find something appropriate.

    c. develop an awareness of the vocational and avocational aspects of mathematics.

2. Provide for each pupil a basis for continuous growth:
    a. provide experiences that enhance concept formation, develop skills, and utilize mathematics.

    b. provide experiences that lead students from the physical (or concrete, including manipulative devices, etc.,) to the symbolic (or theoretical).

    c. motivate students to learn more mathematics and to desire continuation of their mathematics education.

    d. provide many successful experiences for students so that they will develop a positive attitude towards the study of mathematics.

    e. emphasize the SI (System International) units of measurement (without neglecting important customary units).

    f. develop mental mathematics skills and skills of estimating solutions to problems.

3. Develop the notion that mathematics deals with the investigation and application of ideas:
    a. encourage verbal communication.

    b. provide opportunities for developing a vocabulary of necessary mathematical terms and symbols.

    c. make a conscious effort to develop mathematics reading skills.

    d. provide opportunities for students to explore and to develop their mathematical curiosity.

e. provide experiences for a student to apply known facts to a new problem situation.

f. provide experiences that are sequential in nature so that the pupils progress from known to unknown mathematical ideas.

4. Develop the relationship of mathematics to our culture:

   a. provide examples of the aesthetic aspects of mathematics in nature, art, etc.

   b. develop an awareness of the contributions of mathematics to civilization and its potential role in the future.

Problem Solving

Problem solving is a critical goal of the mathematics curriculum and as such, it must occupy a prominent position in the education of our students. To solve a problem is to find an appropriate response to a situation which is new and unique to the problem solver.

Various methods of approaching problems have been suggested. In general, they may be classified into four stages commonly associated with problem solving: 1)understanding the problem, 2) devising a plan to solve the problem, 3) carrying out the plan, and 4) looking back or reviewing the problem and solution. The appropriateness of any method must be determined by the situations and the nature of the learner. Table 1 on page 16 suggests various methods under each stage. Examples and elaboration of these stages and methods follow.

Understanding the Problem.

Sometimes students will not be able to solve a problem because they do not know the meaning of the terms used, see the relationships in the problem, or understand the question. The methods listed under "understanding the problem" in Table 1 will focus the students' attention on the necessity of knowing the meaning of terms, seeing relationships, and understanding the question. For example, in Problem 1, students might be asked to read the problem aloud, tell the problem to

Table 1

Problem-Solving Behaviors

Understanding the Problem

- Read the problem aloud
- Use resources such as text, dictionary to clarify meaning of terms
- Tell the problem to someone else
- Act out the problem
- Underline or write down only the necessary data
- Determine whether data is sufficient, consistent, reasonable
- Draw a figure
- Make a physical model
- Draw a diagram
- Organize a list
- Make a table
- Make a graph
- Assign a time sequence to the problem
- Outline the problem
- Introduce suitable notation
- Brainstorm on given information, immediate inferences, assumptions
- List all information given and related facts
- State the conditions of the problem
- State or write the problem using the necessary information in a concise form
- Write algebraic equations
- Determine the nature of your answers by trial and error, estimating, etc. (What label will it have? What will the answer look like? How large is your answer?)

Devising a Plan to Solve the Problem

- Guess
- Experiment
- Compute
- Restate the problem
- Use resources, e.g., dictionary, atlas
- Look for a similar problem solved before
- Organize data into list, tables, figures, diagrams, etc.
- Look for patterns in the data (e.g., study tables, lists, figures, or diagrams)
- Work the problem backwards
- Make and verify conjectures
- Generalize the data
- Make deductions
- Check your assumptions
- Specialize the problem (simple sample problems)
- List all possible alternatives
- Write a mathematical sentence
- Break the problem into parts
- Go back to the definition
- Brainstorm

Table 1

Problem-Solving Behaviors

Carrying Out the Plan

Keep the problem continuously in mind

Write and solve an example of the problem

Check each step of the solution to see that the solution process makes sense

Differentiate between reasonable and absurd, logical and illogical

Demonstrate or explain that each step is correct

Use a new plan, if necessary

Looking Back: Reviewing The Problem and Solution

Check your result and process (argument):

> Is the question in the problem answered?
>
> Is your answer within reason?

Apply another technique to solve the same problem

Organize (outline) the approach (steps) used to solve the problem

See your solution at a glance

Explain why and how the approach worked

Extend the problem

Generalize your solution process

Identify key elements of the problem

Apply the process used to solve similar problems someone else, list all information given, and determine the nature of the answer by estimating.

Problem 1. *Moki's brother, David, weighs 140 pounds. His sister Lei weighs 98 pounds. Moki weighs 24 pounds less than his brother and 18 pounds more than his sister. What is the average weight of Moki, Lei, and David?*

In Problem 2, a student might read the problem aloud, tell the problem to someone else, act out the problem, and make a diagram.

Problem 2. *If a recipe calls for 2/3 cup of shortening, how much shortening will be needed for 2-1/2 times the recipe?*

In Problem 3, students might be encouraged to make a physical model; to draw a figure; introduce suitable notation; list all information given; brainstorm on given information, immediate inferences and assumptions; write algebraic equations; and state the conditions of the problem.

Problem 3. *Using Cartesian coordinates, show that the four diagonals of a parallelepiped have a common point which is the mid-point of each.*

In Problem 4, students might be encouraged to act out the problem; make a table; and brainstorm on given information, immediate references, or assumptions. For example, do we assume that no matter how many pieces of candy we buy the rate will remain "two for 15¢"?

Problem 4. *If two pieces of candy cost 15¢, how much would 8 pieces cost?*

Devising a Plan to Solve the Problem. For many students this is the most difficult stage in problem solving. Students need to have choices on what to do to solve problems. While some students seem to have a wealth of ideas on what to do to solve a problem, others seem to be totally lacking in ideas. The methods listed under "devising a plan to solve the problem" in Table 1 are those used by successful problem solvers. Possible instructional strategies the teacher might use to teach these include modeling the technique, providing students with a checklist of choices, asking questions that suggest choices such as: "Have you solved a problem like this before?", "Would it help to draw a diagram?", and discussing possible techniques with the students. For example, given Problem 5 below,

> *Problem 5.* *Three girls on a basketball team weigh 375 pounds. If the heaviest girl lost 15 pounds she would still weigh 15 pounds more than the lightest girl. If the lightest girl gained 15 pounds, then two girls would weigh the same. How much does each girl weigh?*

students might be asked to review a checklist of possible choices and then to select those they feel are appropriate. Some might choose to guess and then check by computing, others might choose to make a diagram or a table.

For Problem 3, previously stated on page 18 students might choose to generalize the data, specialize the problem, look for a similar problem solved before, go back to the definition, or organize the data into diagrams.

Carrying Out the Plan. Devising a plan and carrying it out are closely related. Generally a student would not choose a plan that he/she could not carry out. This implies that teachers must provide students with the "do it" skills if they are to choose alternative plans to solving a problem. The "do it" skills include using a diagram, making a physical model, using guesses, using a table, and using computation. The following examples illustrate several "do it" skills necessary to solve problems.

The "do it" skill in Problem 6 is using a diagram.

Problem 6. *What is the largest number of pieces you can cut a pizza into if you can use only 4 straight cuts?*

The "do it" skills in Problem 7 are making a table, starting with a simpler case of the problem, and looking for patterns.

Problem 7. *A book has 250 pages. How many times will "2" appear when numbering the pages?*

For Problem 3, previously stated on page 18, the "do it" skills include drawing a figure, generalizing the data, going back to the definition, etc.

In carrying out the plan, students should be encouraged to keep the problem continuously in mind. This might be done by having students check each step of the solution to see that the solution process makes sense, write and solve an example of the problem, or demonstrate or explain that each step is correct.

Encourage students to use a new plan if it becomes necessary. Sometimes students will continue to use a technique even when it yields no positive results. For them, looking at a new plan might be more fruitful.

Looking Back: Reviewing the Problem and Solution. This stage of problem solving is frequently overlooked. The behaviors in this stage are as important as the other stages. The first part of this stage involves checking your result and process. Questions such as "Is the question in the problem answered?" and "Is your answer within reason?" should be asked. A good way to have students check their results is to have them apply a different technique to solve the same problem. Other activities to help students check their solutions and processes include having them outline the steps used to solve the problem and explain orally how and why the approaches worked. Encourage students to "see at a glance" the solutions to their problems.

The second part of this stage extends the solution and process to other problems. Students should be encouraged to identify key elements and processes of a problem and to apply them to solve similar problems. For example, after solving Problem 8,

Problem 8. *Seventy-six persons voted for one or the other of two candidates for class president. The winning candidate had 10 more votes than the losing candidate. How many votes did each candidate have?*

students should identify the key elements of the problem and discuss the technique of writing algebraic equations or mathematical sentences. They should then attempt to apply the technique to similar problems as in Problem 9.

Problem 9. *Leilani found ten coins, all nickels and dimes, with a total value of seventy cents. How many dimes did she find? How many nickels?*

Another activity that is helpful in focusing students' attention on key elements and processes of a problem is to have them write problems that are extensions of the original problem. They could develop another problem that has the same solution as the original problem or they could change the conditions of a problem and note what effect that has on the solution. Students could also discuss and solve each others problems.

The problem-solving behaviors discussed under the four stages of problem solving can be taught in the classroom utilizing various instructional strategies. Following is a discussion of these.

Instructional Strategies for Promoting Problem Solving Behaviors.

The following instructional strategies for promoting problem solving behaviors can be utilized in the mathematics classroom: exposure, checklist, behavior teaching, imitation, and teacher questioning. The effectiveness of these strategies is highly dependent on the teacher. He/she might use a combination of these instead of any one in particular.

Exposure. In this strategy the student is given a problem to solve. No instruction is provided to the student regarding problem solving tactics. The student is free to explore whatever problem solving approach he/she chooses. Underlying this strategy is the belief that practice in solving mathematical problems will enhance the student's problem solving skill.

Checklist. In this strategy the student is provided a problem to solve without instruction in problem solving methods. When the student encounters difficulty in solving the problem, he/she is provided a checklist of problem solving behaviors to try.

Behavior teaching. In this strategy the teacher teaches methods to apply in solving certain problems before or at the same time that the student is given a problem to solve. The student then proceeds to solve the problem using those methods.

Imitation. In this strategy, students watch the teacher solve many problems. No attempt is made by the teacher to discuss problem solving methods. Students are then expected to apply the observed problem solving techniques to their problems.

Teacher questioning. The teacher in this strategy, whenever possible, asks problem solving oriented problems when helping students solve problems and when discussing the solution of problems. The hope is that students in time will ask themselves such questions when solving problems.

Problem Solving and Learner Objectives. Problem solving being a practical art like swimming or piano playing can only be acquired by imitation and practice. As a result, it is necessary to provide students with as many as possible opportunities to see and become involved in problem solving. A wide variety of problems from all areas of mathematics must be presented so that students experience the behaviors (described under the four stages of problem solving) necessary to solve problems. These behaviors have been incorporated into the Learner Objectives and Comments/ Activities listed in Chapter IV: Curriculum Guidelines, K-12. Statements such as: have students draw a figure, make diagrams, discuss ways, estimate, and check appear throughout the Curriculum Guidelines.

CHAPTER IV

CURRICULUM GUIDELINES K-12

The curriculum guidelines on the following pages provide directions, as they focus attention on classroom instruction, for accomplishing the goals of mathematics education.

The guidelines are organized to highlight the four major goals of mathematics education and are arranged so that specific objectives for a certain grade or topic may be located readily. To accomplish this, the following five subsections have been prepared as outlined below.

Scope and Sequence: An Overview. This section provides by means of two tables a general indication of the grade levels where certain mathematical topics and problem-solving behaviors are taught. In addition, the three types of experiences are indicated and discussed: 1) intuitive; 2) for understanding, skill, and application; and 3) for mastery of skills with understanding and for applying skills and concepts in a variety of situations. This section begins on page 25.

Performance Expectations. This section displays performance expectations for mathematics related to the Foundation Program Objectives (FPO). Listed here are bench-mark expectations for students by the end of grades 3, 6, and 8. For the high school level, performance expectations are listed for the required program and for students who elect to enroll in specialized electives. This section begins on page 30.

Curriculum Guidelines, K-6. This section, (page 37) contains learner objectives for three major mathematical topics: Numbers and Operations, Geometry, and Measurement. Comments and activities are suggested for most objectives in order to bring them in focus. Teachers are encouraged to expand this section with their own comments and activities. Learner objectives that lead to student attainment of the performance expectations are also displayed in this section.

Curriculum Guidelines, 7-8. In this section (page 92), a short description of the mathematics program for intermediate school students is followed by learner objectives along with corresponding comments and activities. These learner objectives are classified according to three important mathematical topics: numbers and operations, geometry, and measurement. Learner objectives that lead to student attainment of the performance expectations are also displayed in this section.

Curriculum Guidelines, 9-12. This section gives an overview of the high school mathematics program with a discussion of alternatives for students according to achievement in mathematics, interest, and need for the course(s). Three options are explained and procedures for moving from one option to another within the regular program are described for pupils whose vocational or educational plans change. Also, in this section is a description of the courses available and a list of learner objectives for some of the courses. This section begins on page 107.

Scope and Sequence: An Overview.

This section summarizes information about the major mathematical topics: Numbers and Operations, Geometry, and Measurement, and about problem solving by means of two tables. These describe school programs available to all students.

Table 2 classifies the mathematics program into three major topics and problem solving with a general indication of grade placement. Table 3 parti-

Table 2

Scope and Sequence: Major Mathematical Topics and Problem Solving

	K	1	2	3	4	5	6	7	8	9 - 12
Numbers and Operations	o	o X	o X *	o X *	o X *	o X *	o X *	o X *	o X *	o X *
Geometry	o	o	o	o X	o X	o X	o X	o X *	o X *	o X *
Measurement	o	o	o	o X	o X *	o X *	o X *	o X *	o X *	o X *
Problem Solving	o	o	o	o X	o X	o X *	o X *	o X *	o X *	o X *

Legend:

o Intuitive experiences

x Experiences for understanding, skill and application

* Experiences for mastery of skills with understanding and experiences for applying skills and concepts in a variety of situations

Table 3

Scope and Sequence: Subdivisions of Major Mathematical Topics and Problem Solving

	K	1	2	3	4	5	6	7	8	9-12
Numbers and Operations										
Whole Numbers										
Forms Concepts	o	o	o ×	o ×	o × ★	× ★	× ★	★	★	★
Adds	o	o ×	o × ★	× ★	× ★	★	★	★	★	★
Subtracts	o	o ×	o × ★	× ★	× ★	★	★	★	★	★
Multiplies	o	o	o ×	o × ★	o × ★	o × ★	× ★	★	★	★
Divides	o	o	o ×	o ×	o × ★	o × ★	o × ★	× ★	★	★
Decimals										
Forms Concepts			o	o	o ×	×	× ★	× ★	★	★
Adds				o	o ×	×	× ★	★	★	★
Subtracts				o	o ×	×	× ★	★	★	★
Multiplies					o	o ×	o × ★	× ★	★	★
Divides						o ×	o × ★	× ★	★	★
Fractions										
Forms Concepts	o	o	o	o ×	o ×	o ×	× ★	× ★	★	★
Adds				o	o ×	o ×	o ×	× ★	★	★
Subtracts				o	o ×	o ×	o ×	× ★	★	★
Multiplies					o	o	o ×	o × ★	× ★	★
Divides						o	o	o ×	o × ★	★
Ratio and Proportion					o	o ×	o ×	×	★	★
Percent							o ×	×	× ★	★
Integers							o	o	o × ★	o × ★
Algebra		o	o	o	o	o	o	o	o × ★	o × ★
Trigonometry									o	o × ★
Analytic Geometry							o	o	o	o × ★
Calculus										o
Geometry										
Geometric Figures										
Sorts	o	o	o ×	o ×	×	×	×	× ★	× ★	× ★
Recognizes	o	o	o ×	o ×	×	×	×	× ★	× ★	★
Identifies and Names		o	o	o ×	o ×	×	×	× ★	× ★	× ★
Draws and Constructs	o	o	o	o	o	o	o ×	×	× ★	× ★
Classifies by Properties			o	o	o	o ×	o ×	o × ★	o × ★	o × ★
Geometric Relationships										
Organizes into Deductive Systems								o	o	o × ★
Measurement										
Time	o	o ×	×	× ★	× ★	★	× ★	★	★	★
Temperature	o	o ×	×	× ★	★	★	★	★	★	★
Linear	o	o ×	o ×	o ×	o ×	o ×	o ×	o × ★	× ★	★
Area	o	o	o	o ×	o ×	×	×	× ★	× ★	★
Volume	o	o	o	o ×	o ×	o ×	o ×	×	× ★	★
Mass	o	o	o	o	o ×	o ×	×	× ★	★	★
Capacity	o	o	o	o	o ×	×	×	× ★	★	★
Graphing	o	o	o ×	o ×	× ★	× ★	× ★	× ★	× ★	★
Statistics			o	o	o	o	o	o ×	o × ★	× ★
Problem Solving										
Data Relationships										
Relates Data to Physical Models and Pictures	o	o	o	o × ★	× ★	× ★	× ★	× ★	× ★	× ★
Relates Data to Mathematical Sentences			o	o	o	× ★	× ★	× ★	× ★	× ★
Uses Inductive Procedures	o	o	o	o	o ×	o ×	o × ★	o × ★	o × ★	o × ★
Organizes and Analyzes Data		o	o	o	o	o ×	o ×	×	× ★	× ★
Draws Conclusions and Checks Them			o	o	o	o	o ×	× ★	× ★	× ★
Uses Deductive Arguments						o	o	o	o ×	o × ★

Legend:

- o Intuitive experiences
- × Experiences for understanding, skill and application
- ★ Experiences for mastery of skills with understanding and experiences for applying skills and concepts in a variety of situations.

tions the three topics and problem solving into their integral elements. For each of these elements a typical grade placement is given and the sequential nature of the elements can be observed.

Both tables show that the development of competence in any area of mathematics is a long-term process covering many years of school. They further focus attention on the spiral nature of instruction with the reintroduction at various grades of a topic through new approaches at higher levels of sophistication. The variety of experiences required for an effective mathematics program is displayed. These experiences include intuitive; those for understanding, skill and application; and others for mastery of skills with extensive applications. Indicated also but in a less specific way is the fact that learning any idea or skill during a given year depends on learning during the previous years.

Different levels of learning experiences are depicted in the tables by the symbols: o, x, *.

The symbols may be understood more completely from the examples which follow.

The meaning of o. The instruction at this level provides for intuitive experiences when learning new concepts. A term often used to describe this stage of instruction is readiness. A few examples are:

1. Students cut regions into equal parts to illustrate fractions.
2. Students fold paper to bisect a segment, draw a right angle with the corner of a card, or use string to approximate an angle bisector.
3. To measure a given distance pupils use available materials such as a pencil, a piece of wire, or a strip of cardboard as a unit segment.
4. In attacking a new problem the pupils apply intelligent trial and error by trying various procedures in an effort to isolate an answer or find a counterexample.

The meaning of x. The instruction at this level is aimed at understanding, skill and application. A few over-simplified examples follow.

1. In learning to multiply 23 x 49, students draw a picture of a rectangle subdivided into four rectangular regions with the area of each related to one of the four products in the algorithm. Later in algebra the students draw a similar picture for (2x +3) (4x +9) to show the four partial products. In each instance, as their understanding develops, the students practice and apply their knowledge.
2. In learning graphing students make a physical model of a bar graph by stacking boxes on their birthday months (marked on the floor). Later they relate data in a table to a graph and draw a variety of graphs: e. g., line, bar, circle. Then with ordered pairs of numbers they make tables, draw, and interpret graphs.
3. The students first encounter the formula a = bc as they study the multiplication facts such as 30 = 5 x 6. Later they encounter unknown factors □ x 6 = 30 for which they think "what number times 6 is 30" before they learn the multiplication property of equality. As a final outcome they understand a = bc as a formula for finding the area of a rectangular region, for finding total cost for a number of items each costing the same, for finding the distance traveled if time and average speed per unit time is given and so on.

The meaning of *. The instruction at this level provides for a wide variety of practice for mastery of skills with understanding. At the same time new applications appropriate for each student as well as problem-solving activities are introduced.

Further information on objectives for developing mathematical competence is contained in the next section on performance expectations and the subsequent three sections describing curriculum guidelines for grades K-6, 7-8, and 9-12.

Performance Expectations.

The Department of Education has developed eight Foundation Program Objectives which, if accomplished, would produce the desired outcomes--an educated person. These objectives form the basis for all disciplines working together to plan, implement, and evaluate the overall instructional program.

Mathematics has a major responsibility in accomplishing two of these Foundation Program Objectives:

Objective I: Develop basic skills for learning and effective communication with others.

Objective III: Develop decision-making and problem-solving skills at the student's proficiency level.

To a lesser extent mathematics is responsible for accomplishing the remaining Foundation Program Objectives:

Objective II: Develop positive self-concept.

Objective IV: Develop independence in learning.

Objective V: Develop physical and emotional health.

Objective VI: Recognize and pursue career development as an integral part of personal growth and development.

Objective VII: Develop a continually growing philosophy that reflects responsibility to self as well as to others.

Objective VIII: Develop creative potential and aesthetic sensitivity.

For Objectives I, III, and VIII, student performance expectations which specify desired behavioral outcomes have been developed for mathematics. Each performance expectation specifies desired knowledge, attitudes, or skills. These are stated in Table 4. The following description is provided for a more thorough comprehension of these performance expectations.

- The expectations for Objective 1, Basic Skills, are listed first. Those related to Objective III, Problem Solving, and those for Objective VIII, Creativity and Aesthetics are listed last.
- The performance expectations are written in the form of demonstrable behavior that requires application of knowledge, skills and attitudes to provide direction and focus for classroom instruction.
- Performance expectations are written for grades 3, 6, and 8. For the high school level, they are categorized as required or as electives. Within the specified grades and for the required and electives categories, performance expectations are clustered in topical areas. The performance expectations within a cluster are arrayed in a progression to assist the teacher in taking the learners from where they are to their fullest potential. Student development must occur in each cluster.
- The increasing sophistication of the mathematics learned as students progress through the grades is evident in the expectations. There are, for example, expectations at grades 6, 8, and high school related to geometric properties. The language indicates growth in ability to deal with these properties in greater depth. Thus, in reading the performance expectations, the idea of appropriateness must be kept in mind.

The related learner objectives for the performance expectations are displayed in the sections that follow.

Table 4

Performance Expectations

Grade 3	Grade 6	Grade 8
• Uses whole numbers and commonly used fractions (e.g., ¼,½) to communicate physical quantities. (How many, how much, etc.) • Recalls the addition and subtraction facts through sums of 18. • Adds and subtracts 3-digit numbers with regrouping (carrying and borrowing). • Recalls the multiplication and division facts through products of 81. • Multiplies 2-digit numbers by 1-digit numbers without regrouping. • Multiplies 2-digit numbers by 1-digit numbers with regrouping. • Divides 2-digit numbers by 1-digit numbers with and without remainders. • Estimates measurements and does arithmetic mentally. • Uses appropriate language e.g., greater than, less than, and equal to in comparing temperatures, masses (weights), lengths, regions (areas), quantities, and times of events. • Tells time to the nearest minute, makes change through one dollar, and takes temperatures to the nearest degree Celsius. • Estimates and measures the length, capacity, and mass (weight) of physical objects using non-standard units.	• Uses whole numbers, decimals, and fractions to communicate physical quantities. • Adds and subtracts whole numbers; multiplies any whole number by a 2-digit number; and divides any whole number by a 1-digit number. • Adds and subtracts like-denominator fractions and commonly used decimals. • Multiplies and divides decimals. • Estimates measurements and does arithmetic mentally. • Uses ratios to compare quantities and characteristics of physical objects. • Adds and subtracts commonly used fractions (mixed and common) with unlike denominators. • Multiplies and divides mixed and common fractions. • Solves simple ratio, proportion and percent problems. • Estimates and measures length, capacity, and mass (weight) of physical objects using standard units including the metric units. • Reads and writes time, money expressions, and temperatures. • Estimates and measures angles, regions (areas), and volume using standard units, including the metric units.	• Does simple arithmetic mentally (e.g., recognizes complements of 100, multiplies and divides by powers of ten). • Adds, subtracts, multiplies and divides decimals. • Adds, subtracts, multiplies and divides fractions and integers. • Uses ratios to compare quantities and characteristics of physical objects. • Solves ratio, proportion, and percent problems. • Uses algebraic techniques and describes their relationship to the properties of real numbers. • Estimates and measures angles, regions (areas) and volume using standard units, including the metric units. • Computes measurements using the four basic operations. • Converts within metric units. • Explains the interrelationship of the metric units. • Describes and explains possible uses and misuses of basic statistical measurements. • Computes measurements of various common plane and solid geometric figures.

Table 4

Performance Expectations
(Grades 9-12)

Required	Electives
Adds, subtracts, multiplies and divides fractions and integers. Uses algebraic techniques and describes their relationship to the properties of real numbers.	Uses algebraic tecnhiques and describes their relationship to the properties of real numbers.
Computes measurements of common plane and solid geometric figures. Describes and explains possible uses and misuses of basic statistical measurements. Calculates and interprets statistical measurements from a set of data. Calculates measures of dispersion and correlation of data.	Calculates and interprets basic statistical measurements from a set of data. Calculates measures of dispersion and correlation of data.
Uses correct terminology in describing the properties of plane and solid geometric figures. Explains relationships of the parts of a geometric figure and among geometric figures. Performs and describes geometric transformations. * Describes ways that geometric properties and relationships are organized in a deductive system.	Explains relationships of the parts of a geometric figure and among geometric figures. Performs and describes geometric transformations. * Describes ways that geometric properties and relationships are organized in a deductive system. Organizes geometric properties and relationships into deductive systems.
Graphs and analyzes polynomial, rational, exponential, and logarithmic functions, and solves corresponding equations and inequalities.	Uses concepts from trigonometry and analysis to graph equations and inequalities and discusses these from a theoretical point of view.
States or writes the problem using the necessary information in a concise manner. Solves problems by translating given situations into mathematical sentences, by breaking the problems into parts, or by working the problem backwards. Generalizes the solution process and applies to similar problems.	States the condition of the problem, introduces suitable notations, and determines whether the data is sufficient, consistent, and reasonable. Proves mathematical statements orally and in writing; writes alternate deductive justifications (proofs). Solves different problems using the same mathematical model and extends problems solved.

Table 4

Performance Expectations

Grade 3	Grade 6	Grade 8
• Estimates and measures the length, capacity, and mass of physical objects using standard units, including the metric units. • Reads and writes time, money expressions, and temperatures.	• Measures and computes measurements using the four basic operations. • Explains the interrelationship of the metric units.	
• Identifies and compares plane and solid geometric figures in the environment • Sorts plane and solid geometric figures according to their observed properties. • Identifies, names and draws various plane and solid geometric figures.	• Identifies, names and draws various plane and solid geometric figures. • Classifies plane and solid geometric figures into various subsets using different specialized properties. • Uses correct terminology in describing the properties of geometric figures.	• Classifies plane and solid geometric figures into various subsets using different specialized properties. • Uses correct terminology in describing the properties of geometric figures. • Explains relationships of the parts of a geometric figure and relationships among geometric figures. • Performs and describes geometric transformations.*
• Makes tables and graphs to display and compare measurement data.	• Makes tables and graphs to diplay and compare measurement data. • Makes, reads, and interprets simple graphs, tables and commonly used schedules (e.g.,class and bus schedules).	• Makes, reads and interprets simple graphs, tables and commonly used schedules (e.g., class and bus schedules).
• Clarifies problems by asking questions, making physical models,drawing pictures, organizing a list, or restating the problem. • Solves problems by estimating, experimenting, computing, listing, or looking for patterns. • Demonstrates and explains how a mathematical problem is solved.	• Clarifies problems by making a graph, outlining the problems, or brainstorming on assumptions. • Solves problems by making and verifying conjectures, by organizing data into lists, tables, figures, and diagrams, or by listing all possible alternatives. • Checks correctness of results and processes.	• Clarifies problems by listing all information given and related facts. • Solves problems by making deductions, working simple sample problems, or checking assumptions. • Devises and uses alternate means to solve problems.

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* Geometric transformations--operations such as turning, sliding, and enlarging.

Table 4

Performance Expectations
(Grades 9-12)

	Required	Electives
	• Exhibits curiosity about mathematics by seeking answers to questions such as "What is the value of algebra," "How are integers like whole numbers," "Why is mathematics one of the great branches of knowledge."	• Exhibits a curiosity about mathematics by compiling examples of how the progress of civilization parallels the progress of mathematics.
	• Discusses objectively the value, power and beauty of mathematics.	• Investigates critically the utility, limitations and beauty of mathematics.
	• Is aware of the lag in time historically from the development of mathematical ideas to the application of them.	• Is aware of the relationship of the development of mathematics to other disciplines.

Table 4

Performance Expectations

Grade 3	Grade 6	Grade 8
● Exhibits curiosity about every new mathematical idea and asks questions which clarify, relate or extend them. ● Gives examples of the value of numbers and geometry in interpreting the environment. ● Is aware of the need for numbers in record keeping and communication to enhance civilization.	● Exhibits curiosity about mathematical ideas with questions such as "How (do you know)," "What would happen if (we were not allowed to use numbers for 24 hours)." ● Is aware of the value and power of mathematics for attacking quantitative and space problems. ● Is aware of the need to use mathematical and measurement skills and concepts in coping with the environment to enhance civilization.	● Exhibits curiosity about mathematics by seeking explanations of why algorithms and formulas work, why is that trick or short cut correct or how do you perform that mental computation. ● Compiles examples of the value, power, and beauty of mathematics. ● Is aware of attempts to build mathematical models (not physical models) to help solve real-life problems throughout the growth of civilization. For example, the attempt to "invent" negative numbers.

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Curriculum Guidelines K-6

The learner objectives on the following pages are more than a grade placement chart, more than statements of subject matter objectives, and more than a list of skills and concepts. They are additionally procedures for accomplishing the four broad goals of mathematics education: 1) Develop mathematical competence to function effectively in today's society; 2) Develop understanding of the importance and relevance of mathematics historically and in the world today; 3) Develop ability to think critically and to solve problems; and 4) Nurture intellectual curiosity and promote the desire to continue learning. If the Comments/Activities opposite the Learner Objectives are followed, they become experiences that promote in some way all four of the major goals for learning mathematics.

Learner objectives are displayed by grade level. For each grade learner objectives have been classified under three major divisions of content: Numbers and Operations, Geometry, and Measurement.

Each learner objective is associated with one grade. This does not mean that the mathematical content associated with it is always introduced there for every student. Effective experiences over an extended period of time are necessary for any student to reach a suitable level of competence for each objective.

The language used indicates the student as "doing something." In some cases words such as adds, writes, checks, compares, and so on identifies rather carefully what the student should be able to do. In other cases words such as investigating, searches for, devises own way, and interprets indicate experimentation and decision-making. Such language is included to foster problem-solving. By referring to the section on problem solving (page 15), you will note that the behaviors associated with the different stages of problem solving are suggested by the language used in stating the learner objectives.

Initial introduction of symbolism is generally indicated. The delay in using formal symbolism is shown, for example, with fractions introduced in kindergarten and their symbols found first in grade 2.

Comments/Activities serve two purposes: To clarify the meaning of the learner objective and to give examples of experiences for students not always found in texts.

The Comments/Activities in many cases provide a basis for lessons which help students learn to communicate, learn how to attack problems, and learn to reason with numbers and geometric concepts.

Many of the problem-solving behaviors noted on page 15 are incorporated here. Teachers are encouraged to have students experiment, organize data, make and check guesses, communicate their understanding of problems and procedures, and so on.

KINDERGARTEN

Learner Objectives	Comments/Activities
Is curious about everything new and asks questions which clarify, relate, or extend ideas.	The natural curiosity of students can be fostered by welcoming all questions, praising good questions more than answers, and serving as a model by wondering and questioning in as many situations as possible: "I wonder what would happen if...?" "Do you suppose this has anything to do with...?" "I wonder how...?" "What if...?" "I wonder why...?"

NUMBERS AND OPERATIONS (ARITHMETIC)

Learner Objectives	Comments/Activities
Whole Numbers --Comparison Searches for sets that have as many as a given set, fewer than or more than a given set.	Have students form pairs as they perform the daily activities. They match students to see which table has the most (or fewest).
--Number Names Writes numbers 0-9	
Arranges sets of objects so that each new set has one more or one less member than another set.	Students are arranged in a set. By matching they find a set with one more or one less. They progress to using objects and later locating pictures which illustrate the ideas.
--Addition and Subtraction Uses addition and corresponding subtraction facts through sums of 5.	Facts are first illustrated with a set of 2 students joining a set of 3. Students report the results of their experiments orally with no written record at this time.
Fractions Investigates separating a whole object into two equal parts to produce halves	Have students cut common objects (e.g., fruit, jello, bread) in half. Extend the activity to geometric figures such as squares, circles, and rectangles. Have students determine if they actually have equal pieces.

GEOMETRY

Learner Objectives	Comments/Activities
Geometric Figures	
Searches the environment for examples of basic geometric figures such as line segments, rectangles, triangles, circles, spheres, cylinders, cubes, and cones.	Have students prepare a bulletin board display of examples located, tell the class of those seen at home and point out as many examples as possible in the classroom.

MEASUREMENT

Learner Objectives	Comments/Activities
Time	
Recognizes relative times of events.	Students might tell, for example, about school in the morning, play in the afternoon, to the beach yesterday, and tomorrow is Wednesday.
Describes happenings of events using the words early, late, later, before, after, etc.	Students might discuss, for example, "we went to the beach earlier today than yesterday," "a nap after lunch," "TV before bed."
Money	
Uses up to five pennies to purchase item.	Have students act as clerk and purchaser at a class store with each checking to determine that the correct amount has been paid.
Temperature	
Describes temperature using relative terms: hot, hotter, warm, warmer, cold, and colder.	Students might describe to the class: "My soft drink was colder than the water in the ocean." "It is warmer in the day time than at night."
Mass	
Compares two objects using words such as heavier, lighter, and same as.	With one object in each hand students compare their masses. They report to the class, "The book is heavier than the tablet." Other objects that could be used are odd-shaped beads, clay, or candies.
Length	
Investigates the height or length of two or more objects to determine which is longer (or shorter).	Some students may handle or move objects to compare them; others may observe and make a guess. Have students discuss the advantages of each method.
Investigates the length of two or more pictured objects by careful observation.	Encourage students to invent ways to improve on their observations.

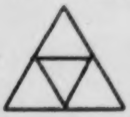
Learner Objectives	Comments/Activities
Ordering Observes and orders up to three objects.	Students should be encouraged to choose their criteria for ordering. They may choose weight, area, capacity, width, etc.
Patterning Recognizes and develops simple patterns involving manipulative objects	Students can use beads, blocks, and other objects to first copy a given pattern and then create their own.

GRADE 1

Learner Objectives	Comments Activities
Is curious about everything new and asks questions which clarify, relate, or extend ideas.	The natural curiosity of students can be fostered by welcoming all questions, praising good questions more than answers, and serving as a model by wondering and questioning in as many situations as possible: "I wonder what would happen if...?" "Do you suppose this has anything to do with...?" "I wonder how...?" "What if...?" "I wonder why...?"

NUMBERS AND OPERATIONS
(ARITHMETIC)

Learner Objectives	Comments Activities
Whole Numbers --Comparison Uses "greater than" and "less than" to compare all numbers up to 10 and for multiples of 10 to 100. (Do not use ">" and "<" at this time.)	Students could use rulers, number lines, and beam balances as aids.
--Counting Observes, verbalizes, and continues patterns in counting by ones, twos, fives, and tens to 100.	As they count using a hundreds board students make observations such as "When I count by fives the numbers end in 0 or 5 (have 0 or 5 in ones place).
--Names and Place Value Displays, reads, and writes the number of tens and ones in any numeral up to 99.	Have students arrange sets of tens and ones in place value charts, with bean sticks or bottle caps in plastic bags and write the corresponding numeral. They reverse the process by observing a number and showing its representation with objects.
Reads, writes, and uses the terms "first, second, third, ... tenth."	Have students use ordinal numbers throughout the day during routine activities. They verbalize "I'm first in line." "I'm tenth from the front."

Learner Objectives	Comments/Activities
--Addition and Subtraction	
Relates addition and subtraction.	Students might tell stories such as "I have 5 cents and I must have 8 cents. How much do I need?" They write 8 - 5 = ___ and think "what do I add to 5 to get 8?" This idea reduces memorization of facts.
Uses addition and corresponding subtraction facts through sums of 10.	Extensive use of physical objects to illustrate addition as joining sets and subtraction as removing a subset is essential with students writing records of their experiments.
Verifies the order and zero property for addition.	Students know this before they come to school: "Our family of 3 joins our neighbor's family of 2. They are the same number if the neighbor's 2 joins our 3." Have students describe or make up similar situations.
Uses relational thinking to discover or rediscover facts.	Describe to the class how you "think" answers. "I know 4 + 4 = 8 so I know 4 + 5 = 9." "I know 8 - 4 = 4 so 8 - 5 is one less or 3." Have students share with others how they "think" answers. Ask students what other sentences they can easily find if you give them the sentence 6 + 4 = 10, etc.
Add three numbers (horizontal or column format) with sums of 10 or less.	Have students describe how they obtain answers. Some may count, others may have discovered they can add in any order, etc.
Fractions	
Recognizes and identifies halves, thirds, and fourths of objects and regions.	Students might cut or fold to contrast two equal parts and two unequal parts. They might color figures in as many different ways as possible so that one-fourth or one-half is colored. (e.g. )

GEOMETRY

Learner Objectives	Comments/Activities
Congruence, Similarity, and Symmetry	
Points out similar, congruent, and symmetric objects (plane and solid) in the environment.	Have students collect objects such as a small and a large heart (valentine) to show similar objects. They show two new pencils as congruent objects and fold paper objects to determine symmetry.

MEASUREMENT

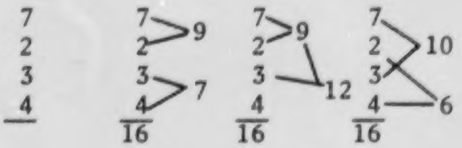
Learner Objectives	Comments/Activities
Time	
Tells time to the nearest hour and ½ hour.	Have students tell time as an integral part of classroom activities. They position the hands of the clock and tell the class the time shown.
Shows awareness of relative movement of the hands on a clock.	Have students actually move the hands on a clock and explain what happens. "The big hand goes around once and the little hand moves from one number to the next."
Money	
Makes change for a quarter using pennies and dimes.	Have students use only pennies and dimes at first to relate making change to the idea of place value. Later include nickels. Have students search for all ways to make change for a quarter.
Temperature	
Describes changes on a thermometer.	Have students take turns reading the daily temperature and reporting to the class. They keep track of indoor and outdoor temperatures for a week.
Mass	
Uses a balance scale to weigh objects to the nearest gram.	Have students look at scales and tell how much objects weigh. Later they guess the weight of an object and check their guess using the scale.
Length	
Measures length using non-standard units.	Students could measure a stick using their pencils. Discuss why students have different measures. Have a contest to see who can find the most different ways to tell the length of an object.
Measures length using rulers marked in centimeters.	Be sure that students use the language "about ___ centimeters long" to build the idea of the approximate nature of measurement.
Area	
Compares regions in the environment using relative terms such as small, smaller, smallest, large, larger, and largest.	For example, a student might say "the grassy part is larger than the sidewalk."

Learner Objectives	Comments/Activities
Volume Investigates which container holds more (or less).	Have students fill and pour contents of different containers to discover which "holds more." Some might arrange the containers according to how much they hold. Have students verify the arrangements.
Graphing Makes a bar graph to display and assimilate data.	Write the months January through December on the floor. Then have students pile their boxes on their birthday month.

Learner Objectives	Comments/Activities
Is curious about everything new and asks questions which clarify, relate, or extend ideas.	The natural curiosity of students can be fostered by welcoming all questions, praising good questions more than answers, and serving as a model by wondering and questioning in as many situations as possible: "I wonder what would happen if...?" "Do you suppose this has anything to do with...?" "I wonder how...?" "What if...?" "I wonder why...?"

NUMBERS AND OPERATIONS (ARITHMETIC)

Learner Objectives	Comments/Activities
Whole Numbers	
--Comparison	
Orders numbers to 100.	Have students group as tens and ones to compare two numbers. Later they verbalize "I know 82 is greater than 79 because it has more tens."
--Counting	
Extends counting to at least by threes and fours to 100.	Have students turn over tags on the hundreds board to show counting by threes or fours.
--Names and Place Value	
Investigates the meaning of 3-digit numerals by showing the number of hundreds, tens, and ones. Also reads and writes 3-digit numerals.	Have students show the meaning of smaller 3-digit numerals with grouped materials. Later they generalize 256 as 2 hundreds, 5 tens, and 6 ones and 256 = 200 + 50 + 6.
Reads, writes, and orally states number names through "twenty."	Students can take turns taking juice, milk, or lunch counts and recording the numbers.
Reads, writes, and uses the terms "eleventh, twelfth, . . . twentieth."	Have students count and record names and positions of fellow students and find out who was twentieth, first, etc.
Recognizes and writes different expressions for the same number.	Have a contest. Who can write the most names for 6 in one minute? This is a good activity at the beginning of a period.
--Number Properties	
Discovers special arrangements of objects for even and odd numbers.	Have students attempt to make geometric shapes such as rectangles and triangles using a particular number of objects. Have them share with the class what they observe about the number (even or odd) of objects and the shapes they can make.

Learner Objectives	Comments/Activities
--Addition and Subtraction Pictures addition and subtraction on a number line.	Have students investigate ways to step off 5 + 3 = 8 with a number line on the floor. Encourage them to invent ways to label a number line to show 20 + 50.
Checks addition by subtracting and subtraction by adding.	Students could do this by using sets of objects. For example, a student might start with a set of 6 and join it to a set of 4 and write 6 + 4 = 10. Next, he/she removes a set of 4 and notes he/she has the original set of 6 and writes 10 - 4 = 6. Students could also argue thus: "I know 6 + 8 = 14 because 14 - 8 = 6."
Discovers and then memorizes addition and subtraction facts through sums of 18.	Use joining of sets as needed. Have students tell how they can rediscover forgotten facts. "10 + 8 = 18 so 9 + 8 = 17." "To add 8 + 5 I do 10 + 3." "To do 15 -7, I think what added to 7 is 15." "To find 14 - 8, I subtract 4 and then 4 again to get 6."
Adds three or more numbers with sums of 18 or less.	Encourage students to invent ways to add columns and describe them to the class.  7 2 3 4 7 2 9 3 4 7 16 7 2 9 3 12 4 16 7 2 10 3 4 6 16
Groups materials and records results for addition and subtraction with carrying and borrowing for two digit numbers, then performs the algorithm without the use of objects.	For early instruction the students write the algorithm as a record of their experiments with materials. Crutches may be used in the algorithm.
Discovers answers for □ in sentences such as 5 + □ = 7, 5 - □ = 2, or □ - 3 = 5 using concrete materials.	For 5 + □ = 7 students should think "I have 5. I must add something to it to get 7." They experiment to find 2 is the only solution. Try 5 + □ = 4 with the class. In mathematics, "It can't be done" is often a correct answer.
Recognizes that zero is the identity for addition and the corresponding subtraction fact.	The word identity is not introduced at this time. The important idea is that 0 added to or subtracted from a number equals that number.

Learner Objectives	Comments/Activities
Verbalizes the idea of the terms: addends, sums, and related subtraction terms.	Technical vocabulary is learned over a period of time. Use short practice periods with questions such as, "What are the addends in 7 - 4 = 3? "How do you know 5 and 3 are addends in 8 - 5 = 3?"
--Multiplication and Division Makes smaller groups, each with the same number of objects from a larger group of objects.	Have students think of division first as equal sharing. They share six pieces of candy with a friend by saying, "One for you, one for me..." and find three in each pile.
Discovers the relationship of multiplication to addition.	Have students arrange objects such as shown and give four interpretations: two 3's are six, 3 twos are 6, twos in 6 are 3, and threes in 6 are 2. xxx xxx
Gives examples of order property for multiplication.	Students might show, for example, that xxxxx is two fives and if rotated xxxxx is five twos xx xx xx xx xx
Understands and uses multiplication facts through products of 25.	The early language of 6 twos are 12 is replaced by 6 times 2 is 12 and x and = are used. Encourage students to ask other students "Show me why 3 x 4 = 12." A variety of answers should be discussed: pictures, counting by ones, counting by fours, etc.
Fractions Recognizes and identifies halves, thirds, and fourths of objects, regions, and set of objects.	Students could cut regions and write 1/2, 1/3, or 1/4 on the parts to show equal parts of the whole. Students should cut some wholes into 4 pieces, some being unequal, and tell why each part is not labeled 1/4.
GEOMETRY	
Sorts plane and solid objects and figures by observing similarities and differences in their properties.	Have students establish their own criteria for sorting. Some possibilities are, "Has five sides," "Has no straight sides."

Learner Objectives	Comments/Activities
--Congruence	
Devises ways to determine figures of the same size and shape.	Have students search for congruent figures in the classroom. Also one student forms a figure on the geoboard; another student forms one congruent to it.

MEASUREMENT

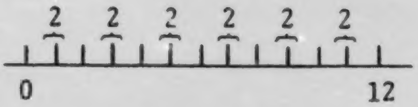
Learner Objectives	Comments/Activities
Time	
Reads and writes time expressions for hours and minutes.	Have the students keep a record of how they spent the morning, recording the times for starting each new activity.
Names the days of the week and months of the year.	Have students keep the classroom calendar and announce to the class, "Today is Tuesday, November 5. "
Makes time schedules for specific activities.	Have students keep a record of times for various classroom activities.
Money	
Names and writes the value of coins through one dollar.	Have one student display a collection of coins and a partner write the value.
Uses symbols for cent and dollar.	Have students list the cost of common commodities like milk, juice, lunch, candy, and Star War cards in two different ways: cents and dollars.
Temperature	
Reads, interprets, and records temperature shown on a thermometer using the degree symbol.	Students can read and record temperature at different times of day. Encourage and assist students in organizing data.
Mass	
Estimates and then uses a balance scales to weigh objects to the nearest kilogram.	Have students find objects that weigh one kilogram. Objects from the classroom might be a number of books, a can of sand or a stack of paper. They weigh each after estimating.
Length	
Estimates and measures (using ruler and meter sticks) to the nearest meter and decimeter.	• Have students locate objects in the classroom that are about one decimeter in length. Do the same outdoors for one meter. • Historical tidbits such as, "The meter unit was once defined as one ten-millionth of the distance from the equator to the North Pole," may help to foster student curiosity and appreciation for the history of mathematics.

Learner Objectives	Comments/Activities
Uses body parts as illustrations of linear units and as a means of visualizing the length of objects.	Have students determine what part of a certain fingernail is about 1 centimeter long or what part of the foot is about 1 decimeter.
Area	
Compares the region of two geometric figures stating whether one is larger than, smaller than, or the same size as another.	Have students invent their own ways of comparing. First they compare by physically placing one figure on top of the other. Later when they find objects that can't be moved they compare a tracing of one to the other.
Volume	
Investigates to discover the relationship between a liter and a deciliter.	Have students compare the amount of liquid that a deciliter and liter container hold by filling one with water and then pouring it into the other. Ask, "How many times do you have to do this?" "What part of the larger is the smaller container?"
Estimates, measures, and records to the nearest liter and deciliter.	Have students collect a variety of 1 liter containers to show that size is not always an indicator of the volume it can hold.
Graphing	
Interprets data given in bar graphs.	Display a bar graph in which the number of letters in each student's name is shown; have the students interpret this bar graph and from it make a table.

Learner Objectives	Comments/Activities
Is curious about everything new and asks questions which clarify, relate, or extend ideas.	The natural curiosity of students can be fostered by welcoming all questions, praising good questions more than answers, and serving as a model by wondering and questioning in as many situations as possible: "I wonder what would happen if...?" "Do you suppose this has anything to do with...?" "I wonder how...?" "What if...?" "I wonder why...?"

NUMBERS AND OPERATIONS
(ARITHMETIC)

Learner Objectives	Comments/Activities
Whole Numbers --Comparison Orders numbers to 1,000.	Have students discuss a variety of ways to order numbers. "Which comes first when I count: 79, 57, or 38? If I think about place value, I can tell."
--Counting Counts numbers less than 1,000 by hundreds, tens, fives, fours, threes, twos, and ones.	Have students look for patterns. Ask, "If a number has a 0 in the ones place is it in counting by 10? by 5? by 2?
--Names and Place Value Reads, writes, and orally names numerals through 1,000 and identifies the place value of each digit.	Have students find large numbers in newspapers and magazines; have them read the numbers and state the place value of the digits.
Reads and writes Roman numerals through ten.	You may have students make a clock face with Roman numerals and hunt for places where those numerals are used.
Uses ordinal and cardinal numbers.	• Have students count a set of playthings. Then have the students order them in terms of their preferences. • Have a monitor record the order of students arriving to class.
--Addition and Subtraction Adds and subtracts two 3-digit numbers with regrouping.	For most students even at this stage of development the use of physical materials is necessary before they verbalize the procedures and eventually practice a skill.

Learner Objectives	Comments/Activities
Adds three addends each of three digits or less.	Have students search their environment for examples of the use of column addition; for example, total score for a team during four quarters of football or total attendance at school given number present in each room.
Explores ways of adding and subtracting without paper and pencil.	Encourage students to describe ways they use to find sums and differences mentally.
--Multiplication and Division	
Pictures multiplication and division on a number line.	The intuitive language, 6 twos, provides students with a basis for a number line picture.  Similarly 12 ÷ 2 as twos in 12 give a mental picture of what to draw.
Understands and uses the relationship of multiplication and division.	Students might describe the picture at the right as showing 2 x 5, 5 x 2, 10 ÷ 2, and 10 ÷ 5. Have students check their division by multiplying. xxxxx xxxxx
Groups three factors in any order and finds all products the same.	Encourage students to experiment to find the easiest way to find the results for a multiplication such as 2 x 87 x 5.
Relates multiplication to addition and division to subtraction.	Students might tell stories about equal sharing. For example, "There are eight pieces of candy and each student gets two pieces. How many students can have candy? Two for the first student, so 8 - 2 =6, two for the second, so 6 - 2 = 4, etc.
Uses basic multiplication facts and corresponding division facts through products of 81.	Many students need individual help with these facts. Have students discuss with each other ways to reduce memorization such as "9 sevens = 7 nines" or "6 sevens are 42 so sevens in 42 are 6."
Multiplies with one factor less than 100 and the other factor less than 10.	Students should discover various ways to find, for example, 3 x 21. They may use place value charts or money; they may add or use expanded algorithms prior to writing in the shortcut form.

Learner Objectives	Comments/Activities
Divides with 1-digit divisors and 1 or 2-digit quotients.	Students should first use place value charts to perform 36 ÷ 3 reading it "36 into 3 equal parts." Later they subtract one 3 at a time and then subtract multiples of 3.
Fractions	
Recognizes and identifies two-thirds and three-fourths of objects, regions, and sets of objects.	Students using physical materials show two-thirds as two of three equal parts of 1. Later they find two of three equal parts of sets.
Reads and writes fractions with denominators 2, 3, 4, 5, and 10 for equal parts of objects, regions, and sets.	By actually making equal regions, or parts of an object, students learn the meaning of the fraction symbol.
Uses words "numerator" and "denominator" correctly.	Plan short lessons to review the technical vocabulary. Use a variety of questions. "Which fraction has the smallest numerator: 6/7 or 5/2?" "Write a fraction with a denominator of 5."
--Addition and Subtraction	
Represents fractions with the same denominators in pictures or with objects, manipulates pictures or objects to find sums and differences.	Have students manipulate objects then use symbols to record results. Later they write, for example, "2 eighths + 3 eighths = 5 eighths."

GEOMETRY

Geometric Figures	
Names and draws basic geometric figures (plane and solid) including points, lines, planes, parallel and intersecting lines.	Have students locate many examples of geometric figures. Names such as $\overleftrightarrow{AB}$ for line AB is not introduced at this time.
Geometric Properties	
Investigates the properties of lines, rectangles, triangles, and circles.	Students might discover, for example, that the diameters of a circle always contain its center, that only one line passes through two points but many lines pass through one point, etc.
--Symmetry	
Investigates line symmetry.	Have students guess if figures have line symmetry. Have them verify their guess--paper folding or mirrors could be used. Students could also sort figures using the number of line symmetries as criteria.

Learner Objectives	Comments/Activities
--Congruence Draws figures of the same shape and size.	Students could work in pairs with one drawing a figure and the other trying to make a copy of it. Questionable copies should be verified by the students. Placing one figure on top of the other is one possible way to verify.
Terminology Understands the terms: angle, sides corners, radius, diameter, parallel, and diagonals.	On the overhead projector demonstrate the meaning of diameter by placing an object (e.g., straw, stick) in various positions in a drawn circle.
Angles Locates examples of angles, forms angles, and tests for right angles.	Encourage students to use readily available objects to test for right angles. For example, the corner of a card.

MEASUREMENT

Time Tells and writes time to the nearest minute.	Set the hands of a clock in various positions. Students write the time shown and tell how they determined their answers.
Describes a year in terms of months and number of days.	Have students make a table of the names of the month and number of days per month. Have students tell, "How many months in a year? two years? etc. How many days in a year? two years? etc."
Money Reads and writes money expressions.	Using a catalogue or newspaper, have students tell how much each of their favorite items cost.
Makes change through one dollar.	Have students find all ways to make change if given a dollar for an 84 cents purchase.
Counts a collection of coins and bills and records the amount using dollar sign and decimal point.	Have students, given an amount such as $2.53, show it with the fewest bills and coins.
Temperature Describes the readings on the thermometer for boiling and freezing points of water.	Have students observe when and how the thermometer reading changes in boiling water and in ice cubes. Then have them describe or draw two pictures: one for a reading of boiling point and one for freezing point.

Learner Objectives	Comments/Activities
Mass	
Investigates to discover the relationship between a gram and a kilogram.	• Have students guess and verify with a dictionary the meaning of "kilo." • Have students compare different masses by using a balance beam.
Length	
Finds the perimeter of plane objects, pictures, and plane geometric figures.	Students could identify the meaning of length one on the geoboard and then find as many figures as possible with a perimeter of 6 units. Have them repeat for different units. (Note that it is not possible to show with perimeters of 5, 7, or 9.)
Area	
Explores the meaning of area.	Ask students to determine how large a certain region is by selecting their own unit. For example, students may choose their hand, circular objects, rectangular objects, etc.
Volume	
Estimates, measures, and records to the nearest centiliter and milliliter.	Have students guess the capacity of various containers, e.g., milk cartons, eye droppers, can of soup, perfume bottle, straw, teaspoon. Then have them measure and record to the nearest centiliter or milliliter.
Forms figures with unit cubes and tells the volume of each.	Provide students with a supply of unit cubes. Have them make structures and describe the volume as 5 cubes, 6 cubes, etc. Have students make all possible structures using a particular number of cubes, e.g., 6 cubes.
Investigates the relationship of capacity units and volume units.	Have students find out by using water or sand how many liters one cubic decimeter holds.
Graphing	
Collects, records data, and draws graphs.	A variety of data might be collected. For example, height, number in each ethnic group, types of cars that are parked or go by in 15 minutes.

Learner Objectives	Comments/Activities
Is curious about everything new and asks questions which clarify, relate, or extend ideas.	The natural curiosity of students can be fostered by welcoming all questions, praising good questions more than answers, and serving as a model by wondering and questioning in as many situations as possible: "I wonder what would happen if...?" "Do you suppose this has anything to do with...?" "I wonder how...?" "What if...?" "I wonder why...?"

NUMBERS AND OPERATIONS (ARITHMETIC)

Whole Numbers

--Comparison	
Orders numbers to 100,000.	Have students explain to the class how they decide on the order of two numbers.
--Names and Place Value	
Reads, writes, and orally names numerals to 100,000 and identifies the place value of each digit.	Have students search newspaper and magazines for examples of large numbers and have them tell the class the meaning of each digit in their example.
Reads and writes Roman numerals to 25.	
--Rounding	
Rounds to the nearest thousand.	Have students explain how they round. One effective aid (in most cases) is shown on the right for rounding 1243 to the nearest thousand. A pencil is placed over the tens and ones. Then 12 is closer to ten than 20. Thus the answer is 1,000. 2000 1243 1000
--Addition and Subtraction	
Shows proficiency in adding and subtracting whole numbers.	
Estimates sums and unknown addends by rounding or by performing two steps.	For example, the sum of 36 and 53 is estimated by rounding 36 to 40 and 53 to 50, then adding 40 and 50.
--Multiplication and Division	
Estimates products by rounding.	Discuss with the students the reasons for rounding, e.g., to avoid bizarre mistakes. Also discuss why of several satisfactory estimates one might be considered the best.

Multiplies with 2-digit multipliers.	While less concrete representation is used for these examples understanding is still emphasized. Have students think for 13 x 42 "I need 13 forty-twos. I'll find 10 forty-twos plus 3 forty-twos."
Multiplies numbers by multiples of 10 and 100.	Students could think of multiples 10 and 100 as multiples of ten cents or one dollar.
Divides with one-digit divisor and quotient three digits or less.	• Some students will still profit from physical representation using blocks or place value charts. For 468 ÷ 2 they separate 4 hundreds, then 6 tens, and then 8 ones into two equal parts. • Have students determine how many bags of mangoes were sold in a day if 369 mangoes were sold and each bag holds 9 mangoes. Provide problem solving help by suggesting drawing a figure, acting it out, using physical models, making a table, etc.
Checks a multiplication by division and a division by multiplication.	Have students make oral statements about simpler divisions in order to check. For 14 ÷ 3 they say 4 threes plus 2 should equal 14. As numbers become larger they make similar statements.

Fractions

Reads and writes common and mixed fractions.	While students are learning to operate with symbols physical representation must be continued. With circles and sectors of circles, for example, the meaning of 2 3/4 as 2 + 3/4 is readily illustrated.
Uses mixed numbers in various concrete situations.	Situations may include giving measurements of various objects, describing observations, and making diagrams.
For fractions and mixed numbers draws conclusion about order.	Students could shade bars to show 1/2, 1/3, 1/4, etc. and from these decide which is larger. The number line is also an effective aid.
Finds equivalent fractions in a variety of ways.	For example, students might investigate through drawing diagrams, paper-folding, cutting paper or solids, or recognizing and using number properties.

Learner Objectives	Comments/Activities
--Addition and Subtraction Adds and subtracts fractions having like denominators.	First have students illustrate these operations with objects and pictures emphasizing why numerators are added and why the denominators are not.
Decimals Reads and writes decimals involving tenths.	Have students use place value charts as they study the place value of decimals and relate meanings to those learned with whole numbers. They also relate decimals and fractions.
--Addition and Subtraction Adds and subtracts two or more decimals expressed in tenths and hundredths.	Be sure students see addition and subtraction of decimals as an extension of those operations with whole numbers.

GEOMETRY

Learner Objectives	Comments/Activities
Geometric Solids Investigates the properties of common geometric solids such as cubes, spheres, and cylinders.	Have students count the number of flat or curved faces, number of edges and vertices of various solids.
Geometric Properties	
--Symmetry From a set of geometric figures chooses those with symmetry.	Where possible students should fold figures to determine a line of symmetry. If this is impossible they should make a paper model of the figure to try folding.
Finds ways of recognizing symmetry where it cannot be drawn or objects folded.	For example, intersection of two streets and the human body.
--Congruence Sort geometric figures into sets of congruent ones.	If the problem involves pictures, have students compare figures by making paper models from tissue paper to compare for congruence.
--Similarity Sorts geometric figures into sets of similar ones.	Have students form a figure similar to a given one on the geoboard. Help them discover that a triangle similar to another has sides twice as long, three times as long, etc.

MEASUREMENT

Learner Objectives	Comments/Activities
Time	
Chooses appropriate units of time from seconds to centuries to describe certain events.	Have students make up problem such as "Which unit(s) of time would you use to describe the winning time for the marathon? the time to fly from Honolulu to Kauai? the time from the discovery of the Hawaiian Islands until now?"
States different units of time in relation to each other.	Students may prepare a bulletin board display to show relationships of time. In addition to 60 minutes = 1 hour, they may also write 1 minute = 1/60 hour.
Compares measurements of time.	• If the race started at 2:35 p.m. and ended at 3:15 p.m. on the same day, how long was the race? • Have students decide which is longer: a 2-hour movie or a 90 minute movie; a 3-week trip or a 20 day trip.
Money	
Makes change through five dollars.	Students can take turns buying and selling with a $5.00 bill and the appropriate change of play money.
Uses operations of addition and subtraction involving money expressions.	Students could use a real menu to to figure out a bill for a lunch order.
Temperature	
Compares readings on different types and sizes of thermometers.	
Makes observations about temperatures using Celsius units.	Using a Celsius thermometer have students read the temperature at various hours of the day, in various amounts of shade, in water of varying warmth, and so on.
Mass	
Estimates mass of various objects and verifies them.	Students, as they estimate the mass of objects, should keep a record so it can be compared to the correct mass from reading a scale.

Learner Objectives	Comments/Activities
Demonstrates knowledge of when to use certain units of mass such as gram, kilogram, and milligram.	A student would not use, for example, a kilogram to weigh a stick of gum.
Length	
Devises own shortcuts for finding perimeters of special figures such as rectangles and regular polygons.	Guide students to discover by examining perimeters of different polygons that counting can be replaced by addition to find the perimeter of any polygon and by multiplication for regular polygons.
Uses addition in finding perimeters of polygons.	Students may first measure the lengths of sides of polygons. The idea of adding to find perimeters is more evident if they place the sides of the polygon end to end on a line.
Makes guesses about known distances in kilometers and finds ways to check them out.	Have students suggest a variety of ways to check guessed distances such as odometer on an automobile or a pedometer. If no mechanical device is available they may measure a step and find "5 steps is about 3 meters. "Using this they may check shorter distances.
Area	
Finds areas of regions by using different types of grids.	If graph paper with squares of varying length of sides is not available have students use a square length 1 as a unit area, a square side 2 as a unit area and so on.
Volume	
Investigates to discover the relationship of capacity units (e.g., liter, milliliter) to mass units (e.g., kilogram, gram).	By holding or using a beam balance, have students use equal capacity containers to compare capacity and mass of materials of different densities.
Graphing	
Makes a line graph to display and assimilate data.	A variety of data might be used including weather reports, sports scores, personal, class, and school data.

GRADE 5

Learner Objectives	Comments/Activities
Is curious about everything new and asks questions which clarify, relate, or extend ideas.	The natural curiosity of students can be fostered by welcoming all questions, praising good questions more than answers, and serving as a model by wondering and questioning in as many situations as possible: "I wonder what would happen if...?" "Do you suppose this has anything to do with...?" "I wonder how...?" "What if...?" "I wonder why...?"

NUMBERS AND OPERATIONS (ARITHMETIC)

Learner Objectives	Comments/Activities
Whole Numbers	
--Names and Place Value	
Reads, writes, and orally names numerals to 1,000,000 and identifies the place value of each digit.	Have students explain how they decide the correct oral description of the place value of a digit. For numerals such as 2,700,000 use also the commonly used form, 2.7 million.
Reads and writes Roman numerals to 100.	Have students decode a secret message written in arabic numerals, with a code of Roman numerals matched with alphabetical letters.
--Rounding	
Rounds to the nearest hundred-thousand.	Locate large numbers in newspapers. Students are more likely to see the need for rounding such numbers. Discuss why 2 hundred thousand from 217,649 rounded is easier to "think about."
--Number Properties	
Investigates the characteristics of prime and composite numbers and classifies numbers less than 50 as prime or composite (or neither).	Have students write all the factors of various numbers (up to 50). Have them partition the numbers into subsets. "Those with one factor," "Those with two factors," etc.
Investigates multiples and factors of a number; and expresses a number as a multiple of some number, and expresses a number as a product of factors.	Multiples and factors are often confused. Have reviews for 5 minutes using questions such as, "A multiple of 6?" "How do you know?" "A factor of 12?" "How do you know?"
Determines common multiples and common factors of two numbers.	Have students study the multiplication table and make lists of multiples.

--Addition, Subtraction, Multiplication, and Division

Finds the product of all whole numbers.	Students could work with multiplication magic squares.
Finds averages.	Provide many opportunities for students to gather data and find averages, e.g., test scores, height, handful of marbles.
Checks a division by multiplying and adding.	Have student suggest various forms for recording the check. For $\begin{array}{r} 13 \text{ r}4 \\ 6\overline{)\,82} \end{array}$ some may like the horizontal form 82 = (13 x 6) + 4. Others may prefer a vertical form: $\begin{array}{r} 13 \\ \times\ 6 \\ \hline 78 \\ +\ 4 \\ \hline 82 \end{array}$
Shows why division by 0 is meaningless.	For 5 ÷ 0 some students might try to show by asking the question, "How many 0's in 5?" Some may try subtraction. Others may try to relate 5 ÷ 0 = ___ to 5 = 0 x ___.
Divides by two-digit divisors and expresses the quotient as a mixed number.	Have students first solve everyday problems using concrete materials. For example, "How many pieces of candy for each of 12 students if they share equally? How many if there are fourteen people?"
Understands and uses the inverse relation of multiplication and division to solve equations having unknown factors.	The word inverse is not used at this time. The important idea, for example, is that multiplying by 1/2 gives the same result as dividing by 2.

--Number Properties

Searches for common multiples of two or more numbers.	Have students use a multiplication table or write sets of multiples of the numbers: multiples of 6: 6, 12, 18, ㉔, 30 multiples of 8: 8, 16, ㉔, 32

Fractions

Investigates ways to express fractions in lowest terms.	Have students use objects, pictures, mental calculation and dividing for a number of fractions to decide which method(s) may be most effective for certain fractions.
Orders fractions from least to greatest using "<" and ">."	Have students use pictures and/or the number line to investigate the order of fractions.

Learner Objectives	Comments/Activities
Compares mixed numbers.	Have students use pictures and the number line to make comparisons. Some students might find other more sophisticated ways to compare.
Expresses fractions as decimals to hundredths.	Relate both the fraction and the decimal to the same picture and/or number line.
--Multiplication and Division Demonstrates with pictures or objects the multiplication of mixed numbers and fractions by a whole number.	Using recipes that call for mixed and fractional number amounts of ingredients, have students draw or measure out the amount of ingredients needed to make twice, three times, etc. the recipe.
Decimals --Comparison Orders and compares decimals using "<" and ">."	Students could compare regions using graph paper.
--Names and Place Value Reads, writes, and gives the place value of any digit through ten-thousandths.	Have students orally give place values. Avoid reading "2.1" as "two point one" but rather state as "two and one-tenth." Guide students to see the similarities and differences in the decimal and whole number place values.
--Rounding Rounds decimals to the nearest tenth, nearest whole number, nearest hundredths, or nearest thousandths.	Round could be related to money situations and/or to metric measurements. For example, rounding 2.84 to the nearest tenth becomes finding \$2.84 to the nearest dime and rounding 3.956 to the closest hundredth could be measuring to the closest centimeter.
--Multiplication and Division Multiplies decimals.	Rectangular regions with decimal dimensions could be used to relate the algorithm to a physical model.
Divides decimals with two-digit divisors.	• Have students find out how much one dozen eggs costs if it costs \$1.50 for a tray of 30 eggs. • Ask students how to find the gas mileage if you drove 97.9 miles and used 12 gallons of gas.
Ratio Uses ratio for comparisons of numbers and physical quantities.	Have students, using a piece of string, measure around the base of their thumbs,

Learner Objectives	Comments/Activities
	wrist, necks, and waists. Have students look for relationships in those measurements. Discuss other mathematical relationships in nature.
Searches for and finds ways of obtaining ratios equal to a given ratio.	For example, students might use their knowledge of equal fractions.
Devises own table to display equal ratios.	For the string activity mentioned above, have students make a table of ratios of various parts of the body.
Expresses and responds to ratios in various written and verbal forms.	For example, "2 to 3," "2/3," "two-thirds," "2 out of 3."
GEOMETRY	
Geometric Solids Studies many cubes, devises own way of making a copy, tries it out, and modifies after examining result.	Materials may be selected by students and may include clay, styrofoam, paper, cardboard, sticks, straws, etc.
Polygons Compares number of sides, angles, and diagonals of polygons.	Students could use a geoboard or dot paper to make figures and show all diagonals. Have them explore to complete a table. Number of sides of polygon 3 4 5 Number of diagonals from one corner 0 1 2
	Have students investigate why a carpenter in checking a rectangular frame measures the diagonals.
Sorts polygons by observing similarities and differences in their properties, describes common features of figures in each class using different classifications.	Have students list many ways to sort, such as number of sides, number of diagonals, number of lines of symmetry, number of angles greater than 90°, etc.
Coordinates Locates points on a grid for an ordered pair of numbers and connects points to draw a geometric figure.	Provide students with a grid with objects like a horse, a surfboard, a tent, etc. located at lattice points. Have students describe how to reach the object by going to the right and up only.

MEASUREMENT

Learner Objectives	Comments/Activities
Time Computes with numbers which have been derived from different measurements of time.	Have students determine how many years it has been since Moses led the Israelites from Egypt to Canaan in 1200 B.C.
Money Uses operations of multiplication and division involving money expressions.	• Using newspaper or catalogues, have students find the unit cost of bulk items or the cost of buying the same toy for each of 7 friends. • Have students find out how much each student would receive if each shared equally their earnings for chores completed if they collected a total of 9 dollars, 19 quarters, 23 dimes, and 62 nickels.
Temperature Makes observations about temperatures using Fahrenheit and Celsuis thermometers.	Have students match given situations with appropriate temperature, e.g., boiling → 100° C
Mass Estimates weights of animals, people, and other heavier objects; suggest ways of verifying them and verifies them.	Students may be given a set of five objects. They first arrange them in order from heaviest to lightest and then estimate the weight of each.
Length Locates distances and/or objects in the environment that have lengths about a kilometer, a meter, a decimeter, and a centimeter.	A walk with the students will help establish approximately how great a distance a kilometer is.
Measures to the nearest millimeter.	Students could compile on the bulletin board objects measured in millimeters, locate items in the environment identified by millimeters (35 mm film), and grow plants and measure their heights at regular intervals.
Area Finds more efficient ways to determine areas of regions.	For example, notes symmetry, uses disection.
Volume Experiments to find ways to detemine the volume of a prism.	Students might count cubes needed to "fill" a prism and uncover short cuts such as adding or multiplying.

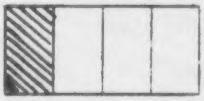
Learner Objectives	Comments/Activities
Graphing	
Interprets data given in a line graph.	Have students make a line graph showing hourly temperatures; interpret rise around room.
Finds actual dimensions of a room and its content and actual distances between known locations from scale drawings.	Have students read local and state maps.
Angles	
Measures angles with non-standard units.	Have students choose arbitrary units to measure with and discuss criteria for the choice of units.

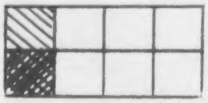
Learner Objectives	Comments/Activities
Is curious about everything new and asks questions which clarify, relate, or extend ideas.	The natural curiosity of students can be fostered by welcoming all questions, praising good questions more than answers, and serving as a model by wondering and questioning in as many situations as possible: "I wonder what would happen if...?" "Do you suppose this has anything to do with...?" "I wonder how...?" "What if...?" "I wonder why...?"

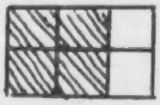
NUMBERS AND OPERATIONS (ARITHMETIC)

Learner Objectives	Comments/Activities
Whole Numbers --Names and Place Value Reads, writes, and orally states numerals to 100,000,000 and identifies the place value of each digit.	By counting or estimating, have students determine the number of letters on a page of a local newspaper. Then estimate the number of pages it will take to get 100,000,000 letters and lay out that number of pages on the floor or board.
Reads and writes Roman numerals and explains the additive and subtractive principles.	Using matchsticks or toothpicks, have students make up equations consisting of Roman numerals and the plus and minus signs, e.g. VI + III = IX. Then have them correct equations that can be corrected by moving exactly one of the sticks, e.g., I - II = I changed to I = II - I.
--Rounding Rounds to the nearest million.	Have students look up populations of each state and round to the nearest million.
--Number Properties Investigates tests for divisibility by 2, 3, 4, 5, and 10.	Have students list observations about divisibility. For divisibility by 2 they see "has 0, 2, 4, 6, or 8 in ones place," "only multiples of 2 are divisible by 2," "if divisible by 4, it is divisible by 2," and so on.

Learner Objectives	Comments/Activities
Expresses numbers as a product of prime factors.	Encourage students to experiment to find various ways to express numbers as a product of primes and to tell advantages and disadvantages of each. Have them make a list of numbers that cannot be expressed as a product of primes.
--Division Uses the division algorithm in finding the quotient of all whole numbers.	Recognize the huge variation in ability to perform division. Some students continue to use some form of the "ladder" algorithm while others use short division quickly with all one-digit divisors.
By rounding, estimates quotients.	In general, too little time is spent on estimation. Have students work in groups and decide ways to estimate. For $23\overline{)427}$ they may think, "23's in 427. So 20's in 100 is 5. Now 20's in 4 hundred is 4 x 5.
Fractions Explores to determine when two fractions are equal.	Have students first check for equality of two fractions by examining objects and pictures, looking at rules and studying a number line. Next have them try multiplying numerator and denominator of one by the same number.
Uses division to express a fraction as a decimal.	These should be limited to terminating decimals, and repeating decimals should be rounded off.
--Addition and Subtraction Adds and subtracts commonly used fractions and mixed numbers with unlike denominators.	At first have students use pictures to avoid silly mistakes. For mixed numbers have students estimate the answer by adding or subtracting only the whole numbers.
--Multiplication Demonstrates with pictures the multiplication of commonly used fractions.	For 1/2 x 1/4 have students fold a paper into 4 equal parts. They shade 1/4 of the paper. They fold the paper in half. They darken 1/2 of the shaded part.





Learner Objectives	Comments/Activities
Decimals	
--Names and Place Value Reads, writes, and gives the place value of any digit through millionth.	Have students make and study place value charts through the millionths. Provide them with the widths of some tiny objects, e.g., water molecule= .00000001 inch, and let them read the number without use of the charts. Then read some numbers to the millionths, and have the students write them out.
--Rounding Rounds decimals to the nearest hundred-thousandths.	Discuss who and when one might round to the nearest hundred-thousandths.
Estimates before performing the basic operations.	Be sure the students appreciate why this is done.
Computes with scientific notation.	Discuss the advantages of such notation, e.g., writing large numbers on a test tube.
Uses division to express a fraction as a decimal.	Have students first express fractions as decimals without dividing, e.g., 1/2 = 5/10 = .5. Students list fractions that can be expressed as decimals easily in this way and those that cannot.
Ratio Uses knowledge of equal fraction to determine equal ratios.	Have students draw and interpret pictures. For 2/3 = 4/6 they may say 4 out 6 □ are shaded. Two out of 3 of these are shaded. The same amount is shaded in each case. 
Percent Understands that percent is a special ratio, namely 1 to 100 and may be symbolized as 1/100, .01, or 1%.	Ask students how % and 100 are alike. If may help in understanding percent as hundredths. Have students interpret newspaper reports: "60% of party went swimming" means if 100 present then 60 went swimming.

GEOMETRY

Polygons Classifies polygons in various subsets using different specialized properties.	E.g., rectangles may be classified as parallelograms.

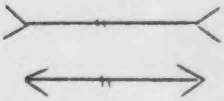
Learner Objectives	Comments/Activities
Classifies and names triangles.	Have students suggest how they might classify triangles. Accept all usable criteria, e.g., number of lines of symmetry.

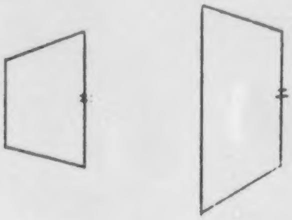
Angles

Learner Objectives	Comments/Activities
Classifies angles as acute, right obtuse, straight, or round.	Draw a figure. Have students name and classify each angle as obtuse, right, or acute.

Construction

Learner Objectives	Comments/Activities
Uses a compass.	Have students draw circles and arcs to complete a design. They color region to make a picture.
Copies a given line segment.	• Given three segments, a ruler, and a compass, have students investigate how to construct a triangle. • Optical illusions! Have students copy one of two equal line segments on tracing paper, then place it over the other to see that they have the same length.





Learner Objectives	Comments/Activities
Bisects a given line segment.	Students could bisect the sides of a triangle and discover geometric relationships.

MEASUREMENT

Time

Learner Objectives	Comments/Activities
Recognizes time zones and reads time schedules.	Have students pretend their friends live in different places on the mainland. Then, using the time zone map in the phone book, have students determine what time it is at their friend's in Tulsa, Oklahoma, etc. Extend by asking them at what time they should call to make sure their friend wasn't sleeping.

Learner Objectives	Comments/Activities
Money	
Relates coins and dollars to decimals.	For example, 5¢ = 0.05
Relates coins to parts of a dollar.	For example, 25¢ = 1/4 of a dollar.
Calculates average cost.	For the average cost of two items have students first find the number half way between. Have students search for situations where average is important (batting average, grades, etc.).
Estimates the total cost of several items.	Have group of students decide ways to estimate and get exact answer mentally. For 57¢ + 83¢ + 24¢ they may add dimes getting 15. Then add cents getting 14. Then add 15 dimes and 14 cents for $1.64.
Temperature	
Recognizes the variability in temperature within a period of days, months, etc. and identifies the range in temperatures within a period of time.	Have students maintain a graph of temperatures at 10 o'clock each school day. For each week/month have them identify range and account for variability, e.g., storm, change of season.
Mass	
Understands the relationship of mass units to capacity units.	Have students weigh a liter of water. Discuss why it may not weigh exactly one kilogram.
Length and Area	
Investigates the possible rectangles that can be formed with a fixed perimeter.	A piece of string or a geoboard could be used. Students should be encouraged to arrange their data in a table: e.g., Perimeter = 8 cm Possible rectangles: 3 cm x 1 cm 2 cm x 2 cm
Investigates the possible number of different perimeters that can be formed with a fixed area.	Have students explore using unit squares. They find, for example, with area 4 perimeters of 8, 10, 12, and 16 are possible (on a geoboard).
Determines experimentally the relationship between circumference and diameter.	Avoid introducing π and formulas at this time. Understanding that the circumference of any circle is about three times its diameter is sufficient here.

Learner Objectives	Comments/Activities
Area	
Experiments to find ways to determine the surface area of a rectangular prism.	Initially have students use rectangular prisms composed of unit cubes so the surface area is in easily countable units. Possible ways that students may discover to find the surface area include counting, counting and adding, counting and multiplying.
For a given situation appropriately finds length, area, or volume.	Have students use a calculator to determine the difference between the edges of two cubes given their volume and the difference of the sides of two squares given their areas.
Volume	
Investigates efficient ways to determine the volume of a rectangular prism.	Short cuts such as adding or multiplying are possible. Students should verify their short cuts by counting unit cubes.
Graphing	
Collects and records data and draws a line graph.	Realistic data from situations known to the students should be used.
Angles	
Estimates and measures angles using a full turn as the standard unit.	Initially have students work with full, half, three-quarters, and quarter turns.

Learner Objectives Related to Performance Expectations

Learner Objectives Related to Performance Expectations

Performance Expectations

Numbers and Operations:

- Uses whole numbers and commonly used fractions (e.g., ¼, ½) to communicate physical quantities. (How many, how much, etc.)
- Recalls the addition and subtraction facts through sums of 18.
- Adds and subtracts 3-digit numbers with re-grouping (carrying and borrowing).
- Recalls the multiplication and division facts through products of 81.
- Multiplies 2-digit numbers by 1-digit numbers without re-grouping.
- Multiplies 2-digit numbers by 1-digit numbers with re-grouping.
- Divides 2-digit numbers by 1-digit numbers with and without remainders.
- Estimates measurements and does arithmetic mentally.
- Clarifies problems by asking questions, making physical models, drawing pictures, organizing a list, or restating the problem.
- Solves problems by estimating, experimenting, computing, listing or looking for patterns.
- Demonstrates and explains how a mathematical problem is solved.
- Is aware of the need for numbers in record keeping and communication to enhance civilization.

Learner Objectives

Kindergarten

Whole Numbers

--Comparison

Searches for sets that have as many as a given set, fewer than or more than a given set.

--Number Names

Writes numbers 0-9.

Arranges sets of objects so that each new set has one more or one less member than another set.

--Addition and Subtraction

Uses addition and corresponding subtraction facts through sums of 5.

Fractions

Investigates separating a whole object into two equal parts to produce halves.

Grade 1

Whole Numbers

--Comparison

Uses "greater than" and "less than" to compare all numbers up to 10 and for multiples of 10 to 100. (Do not use ">" and "<" at this time.)

--Counting

Observes, verbalizes, and continues patterns in counting by ones, twos, fives, and tens to 100.

--Names and Place Value

Displays, reads, and writes the number of tens and ones in any numeral up to 99.

Reads, writes, and uses the terms "first, second, third, ...tenth."

Learner Objectives Related to Performance Expectations

Performance Expectations

Numbers and Operations:

- Uses whole numbers and commonly used fractions (e.g., ¼, ½) to communicate physical quantities. (How many, how much, etc.)
- Recalls the addition and subtraction facts through sums of 18.
- Adds and subtracts 3-digit numbers with re-grouping (carrying and borrowing).
- Recalls the multiplication and division facts through products of 81.
- Multiplies 2-digit numbers by 1-digit numbers without re-grouping.
- Multiplies 2-digit numbers by 1-digit numbers with re-grouping.
- Divides 2-digit numbers by 1-digit numbers with and without remainders.
- Estimates measurements and does arithmetic mentally.
- Clarifies problems by asking questions, making physical models, drawing pictures, organizing a list, or restating the problem.
- Solves problems by estimating, experimenting, computing, listing or looking for patterns.
- Demonstrates and explains how a mathematical problem is solved.
- Is aware of the need for numbers in record keeping and communication to enhance civilization.

Learner Objectives

Grade 2	Grade 3
Whole Numbers	Whole Numbers
--Comparison Orders numbers to 100.	--Comparison Orders numbers to 1,000.
--Counting Extends counting to at least by threes and fours to 100.	--Counting Counts numbers less than 1,000 by hundreds, tens, fives, fours, threes, twos, and ones.
--Names and Place Value Investigates the meaning of 3-digit numerals by showing the number of hundreds, tens, and ones. Also reads and writes 3-digit numerals.	--Names and Place Value Reads, writes, and orally names numerals through 1,000 and identifies the place value of each digit.
Reads, writes, and orally states number names through "twenty."	Reads and writes Roman numerals through ten.
Reads, writes, and uses the terms "eleventh, twelfth, ...twentieth."	Uses ordinal and cardinal numbers.

Learner Objectives

Kindergarten	Grade 1
	--Addition and Subtraction Relates addition and subtraction. Uses addition and corresponding subtraction facts through sums of 10. Verifies the order and zero property for addition. Uses relational thinking to discover or rediscover facts. Adds three numbers (horizontal or column format) with sums of 10 or less.
	Fractions Recognizes and identifies halves, thirds, and fourths of objects and regions.

Learner Objectives

Grade 2

Recognizes and writes different expressions for the same number.

--Number Properties

Discovers special arrangements of objects for even and odd numbers.

--Addition and Subtraction

Pictures addition and subtraction on a number line.

Checks addition by subtracting and subtraction by adding.

Discovers and then memorizes addition and subtraction facts through sums of 18.

Adds three or more numbers with sums of 18 or less.

Groups materials and records results for addition and subtraction with carrying and borrowing for two digit numbers, then performs the algorithm without the use of objects.

Discovers answers for □ in sentences such as 5 + □ = 7, 5 - □ = 2, or □ - 3 = 5 using concrete materials.

Recognizes that zero is the identity for addition and the corresponding subtraction fact.

Verbalizes the idea of the terms: addends, sums, and related subtraction terms.

--Multiplication and Division

Makes smaller groups, each with the same number of objects from a larger group of objects.

Discovers the relationship of multiplication to addition.

Grade 3

--Addition and Subtraction

Adds and subtracts two 3-digit numbers with regrouping.

Adds three addends each of three digits or less.

Explores ways of adding and subtracting without paper and pencil.

--Multiplication and Division

Pictures multiplication and division on a number line.

Understands and uses the relationship of multiplication and division.

Groups three factors in any order and finds all products the same.

Relates multiplication to addition and division to subtraction.

Uses basic multiplication facts and corresponding division facts through products of 81.

Multiplies with one factor less than 100 and the other factor less than 10.

Divides with 1-digit divisors and 1 or 2-digit quotients.

Fractions

Recognizes and identifies two-thirds and three-fourths of objects, regions, and sets of objects.

Reads and writes fractions with denominators 2, 3, 4, 5, and 10 for equal parts of objects, regions, and sets.

Learner Objectives

Kindergarten	Grade 1

Learner Objectives

Grade 2

Gives examples of order property for multiplication.

Understands and uses multiplication facts through products of 25.

Fractions

Recognizes and identifies halves, thirds, and fourths of objects, regions, and set of objects.

Grade 3

Uses words "numerator" and "denominator" correctly.

--Addition and Subtraction

Represents fractions with the same denominators in pictures or with objects to find sums and differences.

Learner Objectives Related to Performance Expectations

Performance Expectations

Measurement:

- Uses appropriate language e.g., greater than, less than, and equal to in comparing temperatures, masses (weights), lengths, regions (areas), quantities, and times of events.
- Tells time to the nearest minute, makes change through one dollar, and takes temperatures to the nearest degree Celsius.
- Estimates and measures the length, capacity, and mass (weight) of physical objects using non-standard units.
- Estimates and measures the length, capacity, and mass of physical objects using standard units, including the metric units.
- Reads and writes time, money expressions, and temperatures.
- Makes tables and graphs to display and compare measurement data.
- Clarifies problems by asking questions, making physical models drawing pictures, organizing a list, or restating the problem.
- Solves problems by estimating, experimenting, computing, listing, or looking for patterns.
- Demonstrates and explains how a mathematical problem is solved.

Learner Objectives

Kindergarten

Time

Recognizes relative times of events.

Describes happenings of events using the words early, late, later, before, after, etc.

Money

Uses up to five pennies to purchase item.

Temperature

Describes temperature using relative terms: hot, hotter, warm, warmer, cold, and colder.

Mass

Compares two objects using words such as heavier, lighter, and same as.

Length

Investigates the height or length of two or more objects to determine which is longer (or shorter).

Investigates the length of two or more pictured objects by careful observation.

Grade 1

Time

Tells time to the nearest hour and half hour.

Shows awareness of relative movement of the hands on a clock.

Money

Makes change for a quarter using pennies and dimes.

Temperature

Describes changes on a thermometer.

Mass

Uses a balance scale to weigh objects to the nearest gram.

Length

Measures length using non-standard units.

Measures length using rulers marked in centimeters.

Area

Compares regions in the environment using relative terms such as small, smaller, smallest, large, larger, and largest.

Learner Objectives Related to Performance Expectations

Performance Expectations

Measurement:

- Uses appropriate language e.g., greater than, less than, and equal to in comparing temperatures, masses (weights), lengths, regions (areas), quantities, and times of events.
- Tells time to the nearest minute, makes change through one dollar, and takes temperatures to the nearest degree Celsius.
- Estimates and measures the length, capacity, and mass (weight) of physical objects using non-standard units.
- Estimates andmeasures the length, capacity, and mass of physical objects using standard units, including the metric units.
- Reads and writes time, money expressions, and temperatures.
- Makes tables and graphs to display and compare measurement data.
- Clarifies problems by asking questions, making physical models drawing pictures, organizing a list, or restating the problem.
- Solves problems by estimating, experimenting, computing, listing, or looking for patterns.
- Demonstrates and explains how a mathematical problem is solved.

Learner Objectives

Grade 2	Grade 3
Time Reads and writes time expressions for hours and minutes. Names the days of the week and months of the year. Makes time schedules for specific activities.	Time Tells and writes time to the nearest minute. Describes a year in terms of months and number of days.
Money Names and writes the value of coins through one dollar. Uses symbols for cent and dollar.	Money Reads and writes money expressions. Makes change through one dollar. Counts a collection of coins and bills and records the amount using dollar sign and decimal point.
Temperature Reads, interprets, and records temperature shown on a thermometer using the degree symbol.	Temperature Describes the readings on the thermometer for boiling and freezing points of water.
Mass Estimates and then uses a balance scale to weigh objects to the nearest kilogram.	Mass Investigates to discover the relationship between a gram and a kilogram.
Length Estimates and measures (using ruler and meter sticks) to the nearest meter and decimeter.	Length Finds the perimeter of plane objects, pictures, and plane geometric figures.
	Area Explores the meaning of area.

Learner Objectives

Kindergarten	Grade 1
Ordering Observes and orders up to three objects.	Volume Investigates which container holds more (or less).
Patterning Recognizes and develops simple patterns involving manipulative objects.	Graphing Makes a bar graph to display and assimilate data.

Learner Objectives

Grade 2

Uses body parts as illustrations of linear units and as a means of visualizing the length of objects.

Area

Compares the region of two geometric figures stating whether one is larger than, smaller than, or the same size as another.

Volume

Investigates to discover the relationship between a liter and a deciliter.

Estimates, measures, and records to the nearest liter and deciliter.

Graphing

Interprets data given in bar graphs.

Grade 3

Volume

Estimates, measures and records to the nearest centiliter and milliliter.

Forms figures with unit cubes and tells the volume of each.

Investigates the relationship of capacity units and volume units.

Graphing

Collects, records data, and draws a bar graph.

Learner Objectives Related to Performance Expectations

Performance Expectations

Geometry:

- Identifies and compares plane and solid geometric figures in the environment.
- Sorts plane and solid geometric figures according to their observed properties.
- Identifies, names and draws various plane and geometric figures.
- Gives examples of the value of numbers and geometry in interpreting the environment.
- Clarifies problems by asking questions, making physical models, drawing pictures, organizing a list, or restating the problem.
- Solves problems by estimating, experimenting, computing, listing or looking for patterns.
- Demonstrates and explains how a mathematical problem is solved.

Learner Objectives

Kindergarten	Grade 1
Geometric Figures Searches the environment for examples of basic geometric figures such as line segments, rectangles, triangles, circles, spheres, cylinders, cubes, and cones.	Congruence, Similarity, and Symmetry Points out similar, congruent, and symmetric objects (plane and solid) in the environment.

Learner Objectives Related to Performance Expectations

Performance Expectations

Geometry:
- Identifies and compares plane and solid geometric figures in the environment.
- Sorts plane and solid geometric figures according to their observed properties.
- Identifies, names and draws various plane and geometric figures.
- Gives examples of the value of numbers and geometry in interpreting the environment.
- Clarifies problems by asking questions, making physical models, drawing pictures, organizing a list, or restating the problem.
- Solves problems by estimating, experimenting, computing, listing or looking for patterns.
- Demonstrates and explains how a mathematical problem is solved.

Learner Objectives

Grade 2

Sorts plane and solid objects and figures by observing similarities and differences in their properties.

--Congruence
Devises ways to determine figures of the same size and shape.

Grade 3

Geometric Figures
Names and draws basic geometric figures (plane and solid) including points, lines, planes, parallel and intersecting lines.

Geometric Properties
Investigates the properties of lines, rectangles, triangles, and circles.

--Symmetry
Investigates line symmetry.

--Congruence
Draws figures of the same shape and size.

Terminology
Understands the terms: angle, sides, corners, radius, diameter, parallel, and diagonals.

Angles
Locates examples of angles, forms angles, and tests for right angles.

Learner Objectives Related to Performance Expectations

Performance Expectations

Numbers and Operations:

- Uses whole numbers, decimals, and fractions to communicate physical quantities.
- Adds and subtracts whole numbers; multiplies any whole number by a 2-digit number; and divides any whole number by a 1-digit number.
- Adds and subtracts like-denominator fractions and commonly used decimals.
- Multiplies and divides decimals.
- Estimates measurements and does arithmetic mentally.
- Uses ratios to compare quantities and characteristics of physical objects.
- Adds and subtracts commonly used fractions (mixed and common) with unlike denominators.
- Multiplies and divides mixed and common fractions.
- Solves simple ratio, proportion and percent problems.
- Is aware of the need to use mathematical skills and concepts in coping with the environment to enhance civilization.
- Clarifies problems by asking questions, making physical models drawing pictures, organizing a list, or restating the problem.
- Solves problems by estimating, experimenting, computing, listing or looking for patterns.
- Demonstrates and explains how a mathematical problem is solved.

Learner Objectives

Grade 4	Grade 5	Grade 6
Whole Numbers	Whole Numbers	Whole Numbers
--Comparison Orders numbers to 100,000.		
--Names and Place Value Reads, writes, and orally names numerals to 100,000 and identifies the place value of each digit. Reads and writes Roman numerals to 25.	--Names and Place Value Reads, writes, and orally names numerals to 1,000,000 and identifies the place value of each digit. Reads and writes Roman numerals to 100.	--Names and Place Value Reads, writes and orally states numerals to 100,000,000 and identifies the place value of each digit. Reads and writes Roman numerals and explains the additive and subtractive principles.
--Rounding Rounds to the nearest thousand.	--Rounding Rounds to the nearest hundred-thousand.	--Rounding Rounds to the nearest million.
--Addition and Subtraction Shows proficiency in adding and subtracting whole numbers. Estimates sums and unknown addends by rounding or by performing two steps.		
	--Number Properties Investigates the characteristics of prime and composite numbers less than 50 as prime or composite (or neither).	--Number Properties Investigates tests for divisibility by 2, 3, 4, 5, and 10.

Learner Objectives

Grade 4

--Multiplication and Division
Estimates products by rounding.

Multiplies with 2-digit multipliers.

Multiplies numbers by multiples of 10 and 100.

Divides with one-digit divisor and quotient three digits or less.

Checks a multiplication by division and a division by multiplication.

Fractions

Reads and writes common and mixed fractions.

Uses mixed numbers in various concrete situations.

For fractions and mixed numbers draws conclusion about order.

Finds equivalent fractions in a variety of ways.

--Addition and Subtraction
Adds and subtracts fractions having like denominators.

Decimals

Reads and writes decimals involving tenths.

--Addition and Subtraction
Adds and subtracts two or more decimals expressed in tenths and hundredths.

Grade 5

Investigates multiples and factors of a number; and expresses a number as a multiple of some number, and expresses a number as a product of factors.

Determines common multiples and common factors of two numbers.

--Addition, Subtraction, Multiplication, and Division
Finds the product of all whole numbers.

Finds averages.

Checks a division by multiplying and adding.

Shows why division by 0 is meaningless.

Divides by two-digit divisors and expresses the quotient as a mixed number.

Understands and uses the inverse relation of multiplication and division to solve equations having unknown factors.

--Number Properties
Searches for common multiples of two or more numbers.

Fractions

Investigates ways to express fractions in lowest terms.

Orders fractions from least to greatest using "<" and ">."

Compares mixed numbers.

Expresses fractions as decimals to hundreths.

Grade 6

Expresses numbers as a product of prime factors.

--Division
Uses the division algorithm in finding the quotient of all whole numbers.

By rounding, estimates quotients.

Fractions

Explores to determine when two fractions are equal.

Uses division to express a fraction as a decimal.

--Addition and Subraction
Adds and subtracts commonly used fractions and mixed numbers with unlike denominators.

--Multiplication
Demonstrates with pictures the multiplication of commonly used fractions.

Decimals

--Names and Place Value
Reads, writes, and gives the place value of any digit through millionth.

--Rounding
Rounds decimals to the nearest hundred-thousandths.

Estimates before performing the basic operations.

Computes with scientific notation.

Learner Objectives

Grade 4	Grade 5	Grade 6
	--Multiplication and Division Demonstrates with pictures or objects the multiplication of mixed numbers and fractions by a whole number. Decimals --Comparison Orders and compares decimals using " " and " ." --Names and Place Value Reads, writes, and gives the place value of any digit through ten-thousandths. --Rounding Rounds decimals to the nearest tenth, nearest whole number, nearest hundredths, or nearest thousandths. --Multiplication and Division Multiplies decimals. Divides decimals with two-digit divisors. Ratio Uses ratio for comparisons of numbers and physical quantities. Searches for and finds ways of obtaining ratios equal to a given ratio. Devises own table to display equal ratios. Expresses and responds to ratios in various written and verbal forms.	Uses division to express a fraction as a decimal. Ratio Uses knowledge of equal fraction to determine equal ratios. Percent Understands that percent is a special ratio, namely 1 to 100 and may be symbolized as 1/100, .01, or 1%.

Learner Objectives Related to Performance Expectations

Performance Expectations

Measurement:
- Estimates and measures length, capacity, and mass (weight) of physical objects using standard units including the metric units.
- Reads and writes time, money expressions, and temperatures.
- Estimates and measures angles, regions (areas), and volume using standard units, including the metric units.
- Measures and computes measurements using the four basic operations.
- Explains the interrelationship of the metric units.
- Makes tables and graphs to display and compare measurement data.
- Makes, reads, and interprets simple graphs, tables and commonly used schedules (e.g., class and bus schedules).
- Clarifies problems by making a graph, outlining the problems, or assumptions.
- Solves problems by making and verifying conjectures, by organizing data into lists, tables, figures, and diagrams, or by listing all possible alternatives.
- Checks correctness of results and processes.

Learner Objectives

Grade 4

Time

Chooses appropriate units of time from seconds to centuries to describe certain events.

States different units of time in relation to each other.

Compares measurements of time.

Money

Makes change through five dollars.

Uses operations of addition and subtraction involving money expressions.

Temperature

Compares readings on different types and sizes of thermometers.

Makes observations about temperatures using Celsius units.

Grade 5

Time

Computes with numbers which have been derived from different measurements of time.

Money

Uses operations of multiplication and division involving money expressions.

Temperature

Makes observations about temperatures using Fahrenheit and Celsius thermometers.

Mass

Estimates weights of animals, people, and other heavier objects; suggests ways of verifying them and verifies them.

Grade 6

Time

Recognizes time zones and reads time schedules.

Money

Relates coins and dollars to decimals.

Relates coins to parts of a dollar.

Calculates average cost.

Estimates the total cost of several items.

Temperature

Recognizes the variability in temperature within a period of days, months, etc. and identifies the range in temperatures within a period of time.

Learner Objectives

Grade 4

Mass

Estimates mass of various objects and verifies them.

Demonstrates knowledge of when to use certain units of mass such as gram, kilogram, and milligram.

Length

Devises own shortcuts for finding perimeters of special figures such as rectangles and regular polygons.

Uses addition in finding perimeters of polygons.

Makes guesses about known distances in kilometers and finds ways to check them out.

Area

Finds areas of regions by using different types of grids.

Volume

Investigates to discover the relationship of capacity units (e.g., liter, milliliter) to mass units (e.g., kilogram, gram).

Graphing

Makes a line graph to display and assimilate data.

Grade 5

Length

Locates distances and/or objects in the environment that have lengths about a kilometer, a meter, a decimeter, and a centimeter.

Measures to the nearest millimeter.

Area

Finds more efficient ways to determine areas of regions.

Volume

Experiments to find ways to determine the volume of a prism.

Graphing

Interprets data given in a line graph.

Finds actual dimensions of a room and its content and actual distances between known locations from scale drawings.

Angles

Measures angles with non-standard units.

Grade 6

Mass

Understands the relationship of mass units to capacity units.

Length and Area

Investigates the possible rectangles that can be formed with a fixed perimeter.

Investigates the possible number of different perimeters that can be formed with a fixed area.

Determines experimentally the relationship between circumference and diameter.

Area

Experiments to find ways to determine the surface area of a rectangular prism.

For a given situation appropriately finds length, area or volume.

Volume

Investigates efficient ways to determine the volume of a rectangular prism.

Graphing

Collects and records data and draws a line graph.

Angles

Estimates and measures angles using a full turn as the standard unit.

Learner Objectives Related to Performance Expectations

Performance Expectations

Geometry:
- Identifies, names, and draws various plane and solid geometric figures.
- Classifies plane and solid geometric figures into various subsets using different specialized properties.
- Uses correct terminology in describing the properties of geometric figures.
- Is aware of the value and power of mathematics for attacking quantitative and space problems.
- Clarifies problems by making a graph, outlining the problems, or brainstorming on assumptions.
- Solves problems by making and verifying conjectures, by organizing data into lists, tables, figures, and diagrams, or by listing all possible alternatives.
- Checks correctness of results and processes.

Learner Objectives

Grade 4

Geometric Solids

Investigates the properties of common geometric solids such as cubes, spheres, and cylinders.

Geometric Properties

--Symmetry

From a set of geometric figures chooses those with symmetry.

Finds ways of recognizing symmetry where it cannot be drawn or objects folded.

--Congruence

Sorts geometric figures into sets of congruent ones.

--Similarity

Sorts geometric figures into sets of similar ones.

Grade 5

Geometric Solids

Studies many cubes, devises own way of making a copy, tries it out, and modifies after examining result.

Polygons

Compares number of sides, angles, and diagonals of polygons.

Sorts polygons by observing similarities and differences in their properties, describes common features of figures in each class using different classifications.

Coordinates

Locates point on a grid for an ordered pair of numbers and connects points to draw a geometric figure.

Grade 6

Polygons

Classifies polygons in various subsets using different specialized properties.

Classifies and names triangles.

Angles

Classifies angles such as acute, right, obtuse, straight, or round.

Construction

Uses a compass.

Copies a given line segment.

Bisects a given line segment.

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WHICH IS ONLY FOR GRADES K-6.

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APPENDIX C

GUIDANCE/COUNSELING FOR BOTH COUNSELORS AND MATHEMATICS TEACHERS AT THE SECONDARY SCHOOL LEVEL

Today, more than ever before, the study and appreciation of mathematics are vital to the intellectual development and to the scientific, industrial, technological, and social progress of society. It is essential that teachers, counselors, supervisors, educational administrators, parents, and the general public work together to provide the best mathematics education possible for all students, regardless of sex, ethnic group, national origin, or ability. All students should be encouraged to keep options open by studying mathematics so as to make maximum use of their talents. Specifically, it is suggested that students include a maximum of mathematics appropriate to their abilities and interest in their high school programs.

The educational, vocational, personal-social choices and decisions made by students should lead to satisfying and worthwhile lives. The important members of the guidance team in each school, both the school counselor and the mathematics teacher, are responsible for helping students gain insight and understanding of themselves and their environment in this decision making. Therefore, they must work cooperatively in:

1. Planning mathematics programs for individual students.
2. Placing students in mathematics courses appropriate to their needs and abilities.
3. Anticipating developments in mathematics and fields that utilize mathematics.
4. Conferring with the school administration with regard to mathematics course offerings.
5. Planning a mathematics program designed for a specific field.

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6. Securing, evaluating, and making available to students a variety of career publications.

7. Planning career-oriented activities.

8. Keeping students informed about:
    a. secondary school and college mathematics programs
    b. vocational and technical school mathematics requirements
    c. college entrance requirements in mathematics
    d. mathematics requirements for majoring in specific areas
    e. procedures for obtaining college credit for mathematics courses taken in high school
    f. career opportunities in mathematics
    g. mathematics needed for specific fields and professions

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